



Art : A. Morreale

α_s from Hadron Structure Phenomenology

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IFPA, Universite de Liege

Workshop on QCD Evolution
May 14-17, 2012, Jefferson Lab

Outline

in collaboration with S. Liuti

- Strong Coupling Constant
 - Perturbative determination
 - Non-perturbative approaches
- Hadron Structure Phenomenology
 - Final State Interaction and Parton Distribution Functions
 - Parton-Hadron Duality
- Non-perturbative QCD coupling from Phenomenology PRELIMINARY

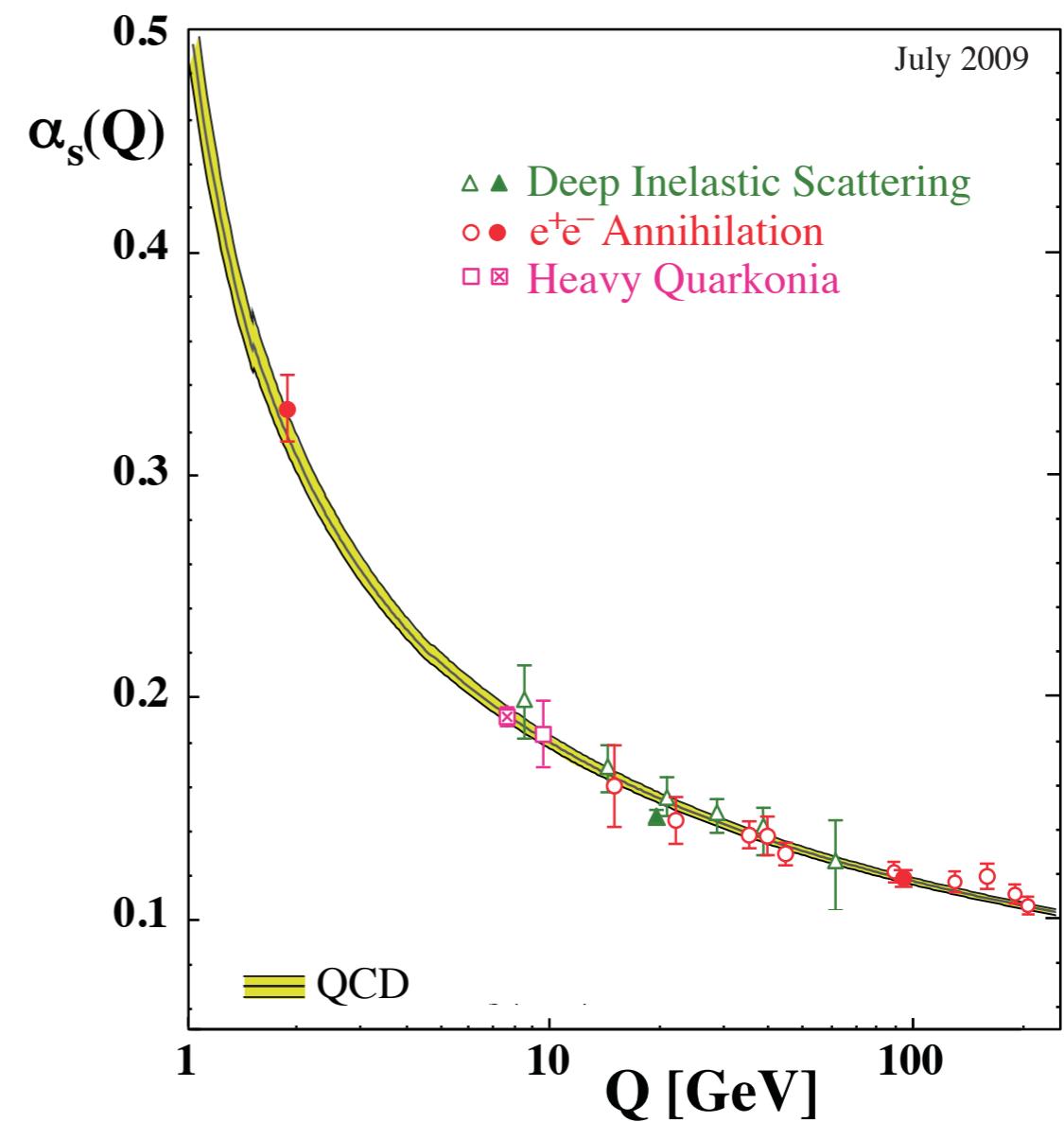
QCD Coupling Constant in pQCD

- QCD with massless quarks
 - no scale parameters
- RGE introduces a momentum scale Λ
 - interaction strength = 1
- Renormalization scheme dependence of Λ
- World data average (2009)

$$\alpha_s(M_{Z^0}) = 0.1184 \pm 0.0007$$

that corresponds to

$$\Lambda_{\overline{MS}}^{(5)} = (213 \pm 9) \text{ MeV}$$



QCD Running Coupling Constant

$$\frac{d a(Q^2)}{d(\ln Q^2)} = \beta_{N^m LO}(\alpha) = \sum_{k=0}^m a^{k+2} \beta_k$$

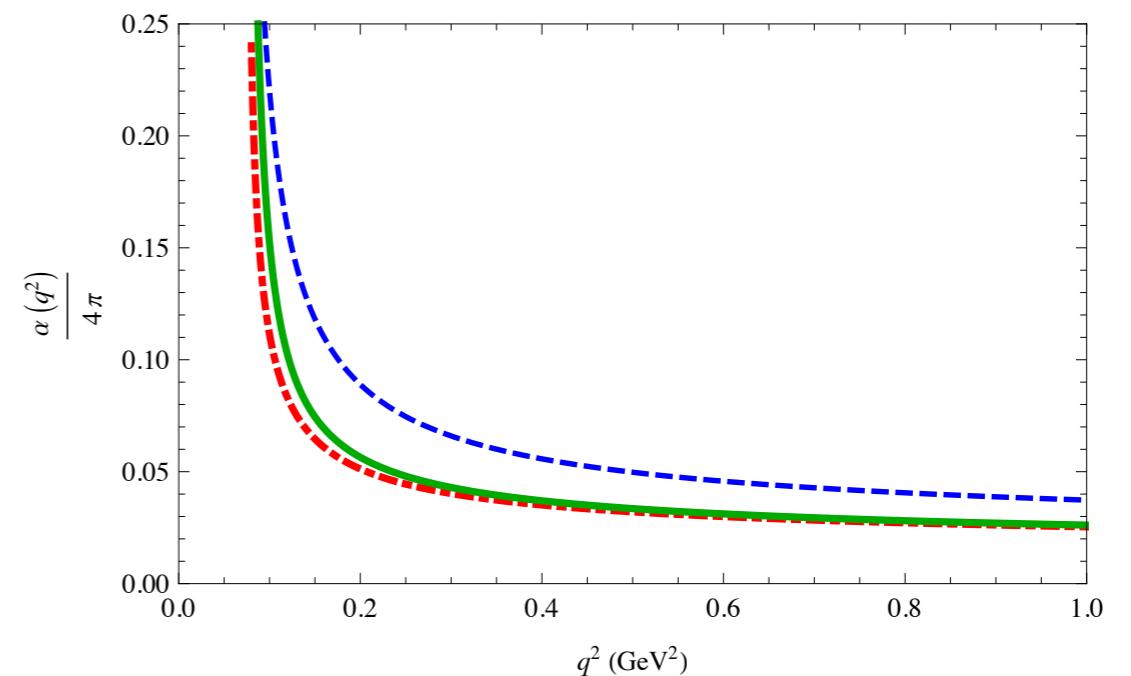
$\overline{\text{MS}}$ scheme

$$a=\alpha_s/4\pi$$

LO exact perturbative solution $\Lambda=250$ MeV

NLO exact perturbative solution $\Lambda=250$ MeV

NNLO exact perturbative solution $\Lambda=250$ MeV



QCD predicts the shape of the running coupling constant, not its value

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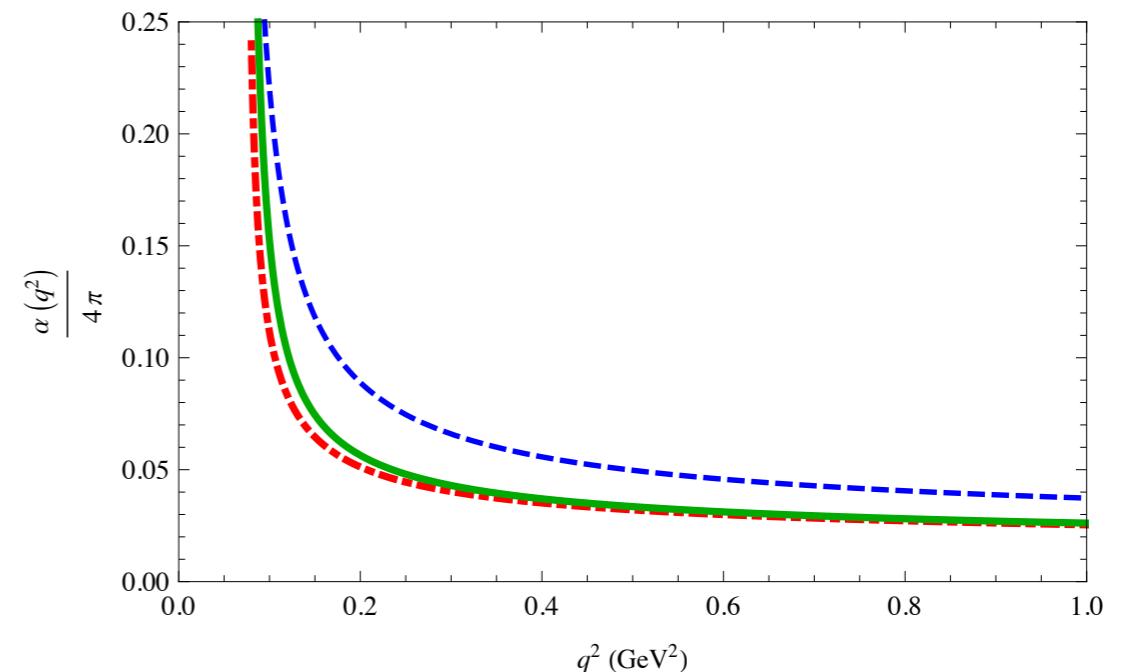
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Intermediate energy?
Perturbative to non-perturbative transition?

Effective Charges

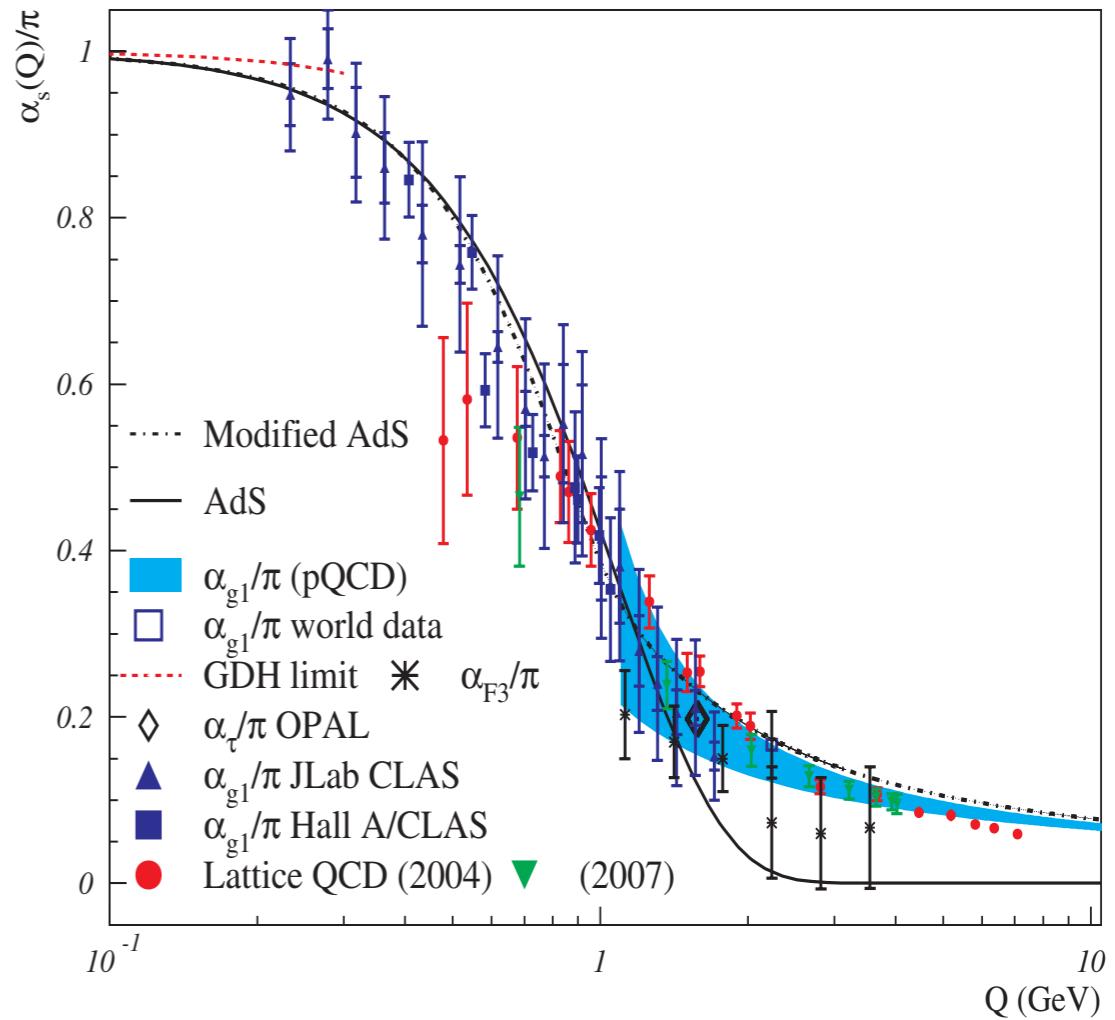
The non-perturbative approach:

- Importance of finite couplings
- Taming the Landau pole

The non-perturbative extraction:

- Effective couplings from phenomenology
- Dimensional transmutation (RG-improved)

→ from RS dependence to Observable
dependence (à la Grunberg)



[Brodsky et al., Phys.Rev.D81]

[Deur et al., Phys.Lett.B60]

Non-perturbative analysis

Qualitative analysis

→ Implications of IR finite α_s in hadronic physics

The non-perturbative approaches:

Cornwall, Phys.Rev.D26, 1453 (1982)

Mattingly & Stevenson, Phys.Rev.D49, 437 (1994)

Dokshitzer, Marchesini & Webber, Nucl.Phys.B469 (1996) 93

Cornwall & Papavassiliou, Phys.Rev.Lett.79, 1209 (1997)

Fischer, J. Phys. G32, R 253 (2006)

Alkofer & von Smekal, Phys. Rept. 353, 281 (2001)

Aguilar, Mihara & Natale, Phys. Rev.D 65, 054011 (2002)

Aguilar, Binosi & Papavassiliou, JHEP 1007, 002 (2010)

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- Cornwall: *gluon propagator*
- Shirkov: *analytic perturbative theory*
- Fischer & Alkofer: *ghost-gluon vertex*

Nonperturbative Gluon Propagator

Solving the Schwinger-Dyson eqs ...

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)
A. C. Aguilar and J. Papavassiliou, JHEP0612, 012 (2006)

$$\Delta^{-1}(Q^2) = Q^2 + m^2(Q^2)$$

$$m^2(Q^2) = m_0^2 \left[\ln \left(\frac{Q^2 + \rho m_0^2}{\Lambda^2} \right) \right]^{-1-\gamma}$$

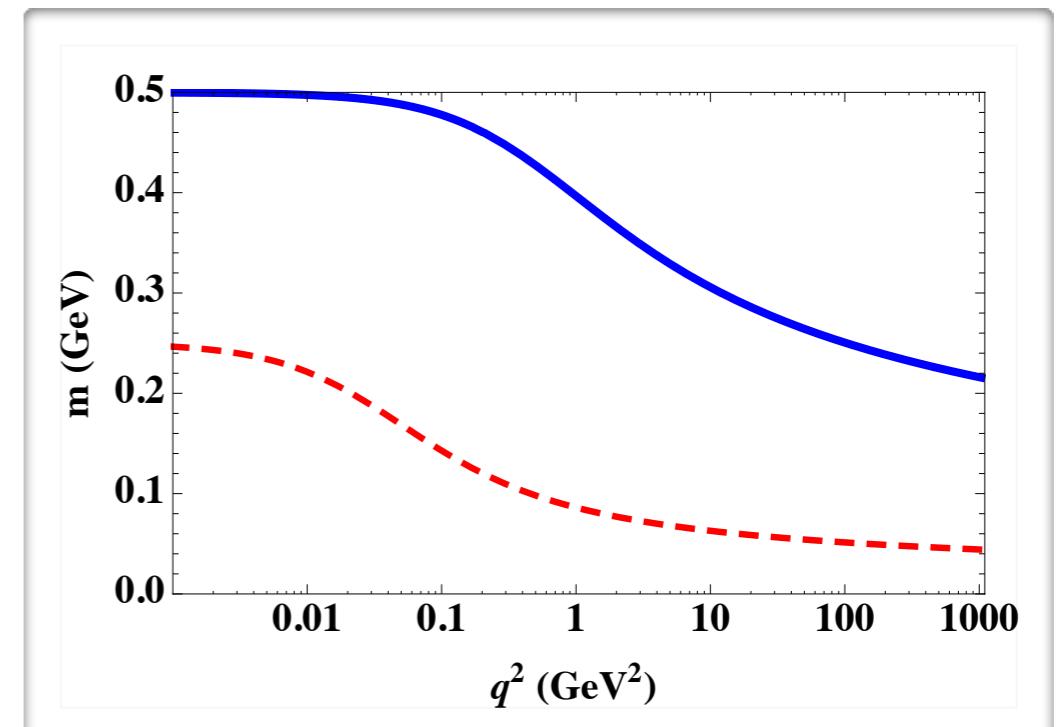
Gluon Mass as IR Regulator

- effective gluon mass
phenomenological estimates

$$m_0 \sim \Lambda - 2\Lambda$$

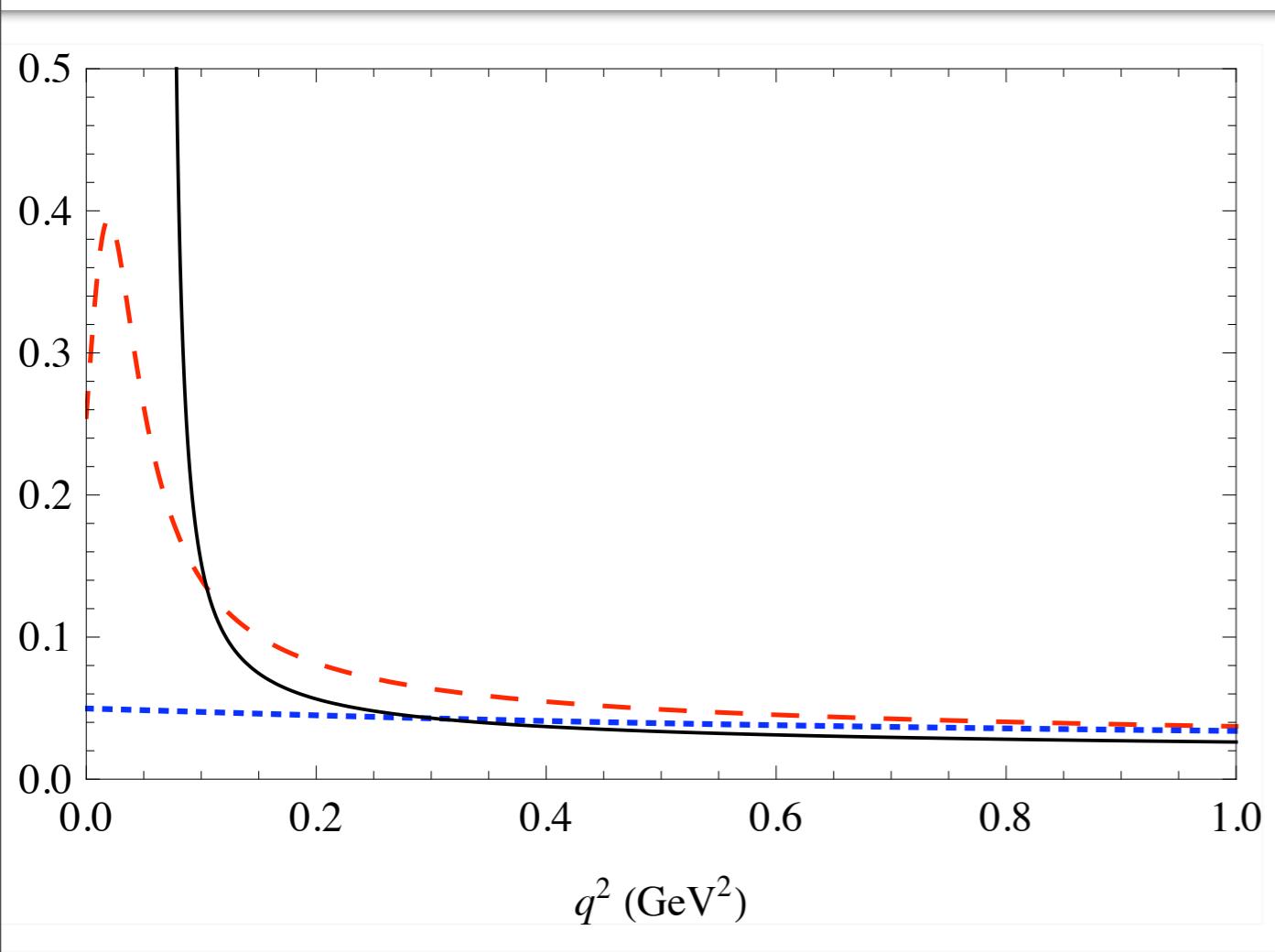
- Solution free of Landau pole
- Freezes in the IR

Low mass scenario
High mass scenario



NP Momentum-dependence of the Coupling Constant

$$\frac{\alpha_{\text{NP}}(Q^2)}{4\pi} = \left[\beta_0 \ln \left(\frac{Q^2 + \rho m^2(Q^2)}{\Lambda^2} \right) \right]^{-1}$$



L0 perturbative evolution
 $\Lambda=250$ MeV ; \overline{MS} scheme

Low mass scenario NP coupling constant
 $m_0=250$ MeV ; $\Lambda=250$ MeV ; $\rho=1.5$

High mass scenario NP coupling constant
 $m_0=500$ MeV ; $\Lambda=250$ MeV ; $\rho=2$.

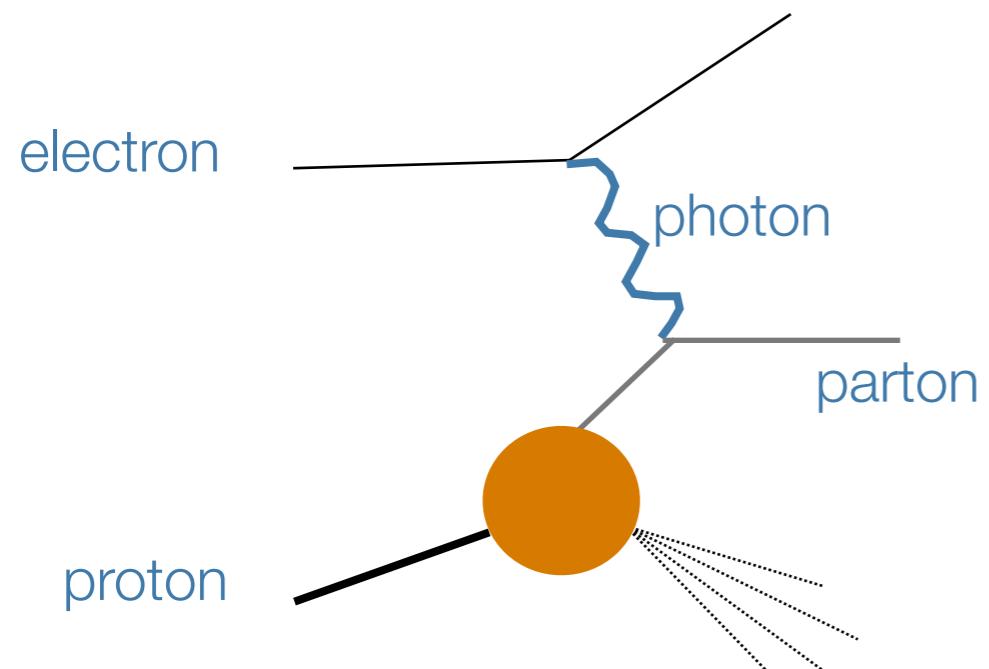
Hadron Structure Phenomenology

Final State Interaction and Parton Distribution Functions

Hard Probes and Factorization

Small size configuration \Rightarrow Hard Probes \Rightarrow Hard processes

Deep Inelastic Scattering



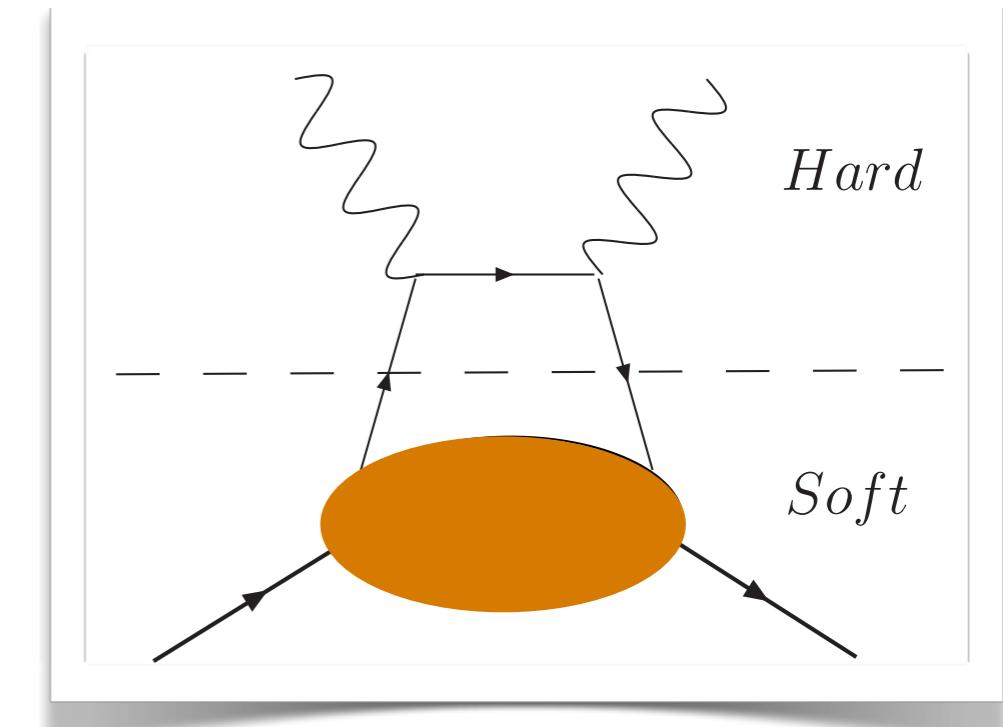
Hadronic tensor \Rightarrow

Parton Model

High energy photon Q^2

Fast-moving proton

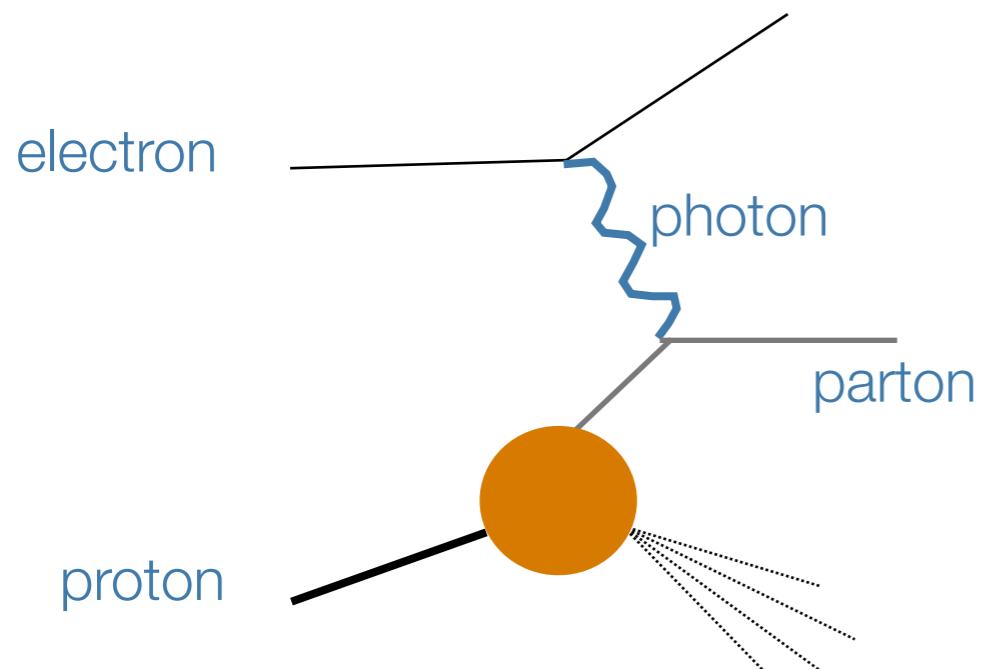
Bjorken scaling



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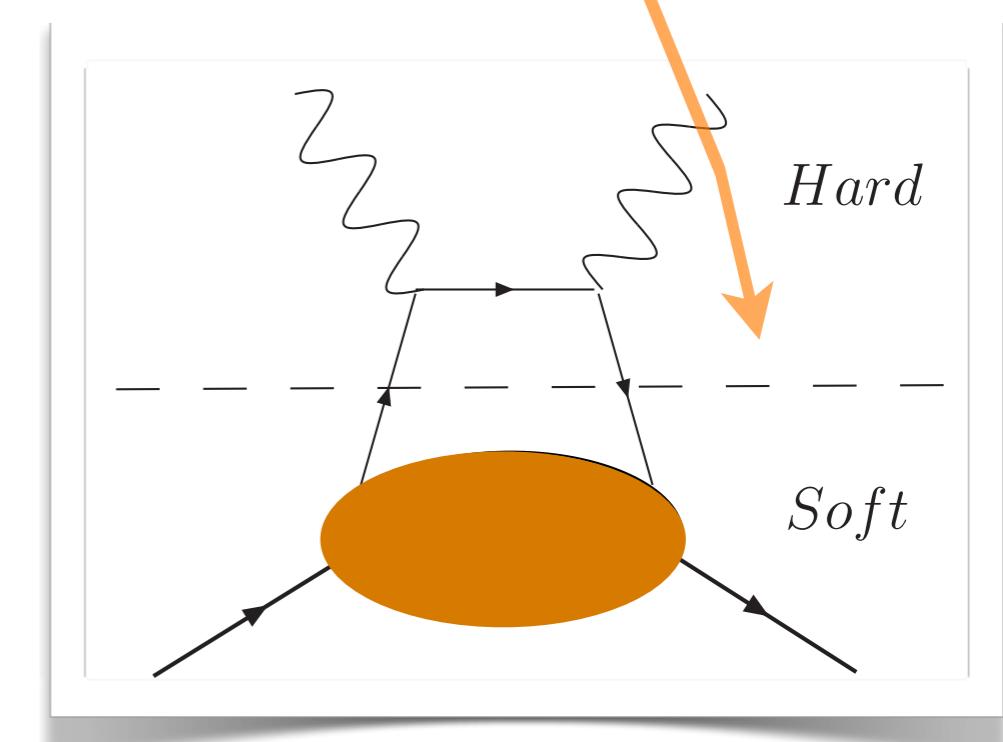
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Bjorken scaling

Factorization
& factorization scale



Transverse Momentum Dependent PDFs

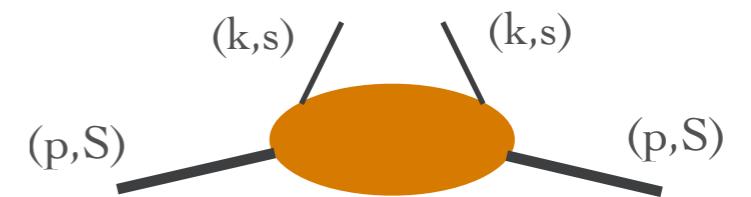
Hadronic matrix elements to $f(x, k_T)$



Number of independent structure functions



Number of Lorentz scalars +hermiticity+parity invariance+Time-reversal invariance



- Relaxing Time-reversal invariance \Rightarrow naive T-odd functions

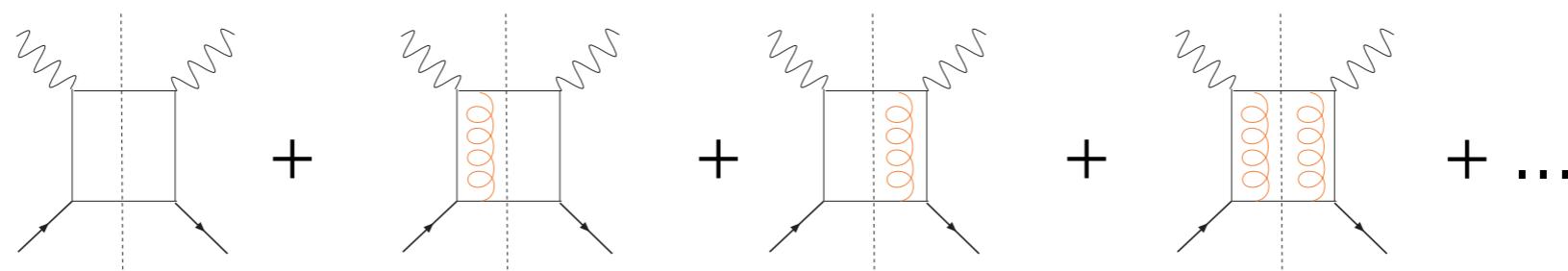
Sivers & Boer-Mulders functions

Sivers, Phys.Rev.D41
Boer & Mulders, Phys.Rev.D57

- Existence of Final State Interactions (FSI) at leading-order

Brodsky, Hwang & Schmidt, Phys.Lett.B530

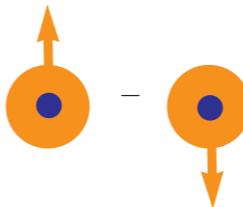
- Importance of the gauge link



T-odd TMDs

The Sivers function $f_{1T}^{\perp Q}(x, k_T)$

⇒ Distribution of **unpolarized quarks** inside a **transversely polarized proton**



The Boer-Mulders functions $h_1^{\perp Q}(x, k_T)$

⇒ Distribution of **transversely polarized quarks** inside a **unpolarized proton**



- Matrix element of low twist operator

$$\begin{aligned} f_{1T}^{\perp q}(x, k_T) &= -\frac{M}{2k_x} \int \frac{d\xi^- d^2 \vec{\xi}_T}{(2\pi)^3} e^{-i(xp^+ \xi^- - \vec{k}_T \cdot \vec{\xi}_T)} \\ &\times \frac{1}{2} \sum_{S_y=-1,1} S_y \langle PS_y | \bar{\psi}_q(\xi^-, \vec{\xi}_T) \mathcal{L}_{\vec{\xi}_T}^\dagger(\infty, \xi^-) \gamma^+ \mathcal{L}_0(\infty, 0) \psi_q(0, 0) | PS_y \rangle + \text{h.c.} \end{aligned}$$

- Importance of gauge link

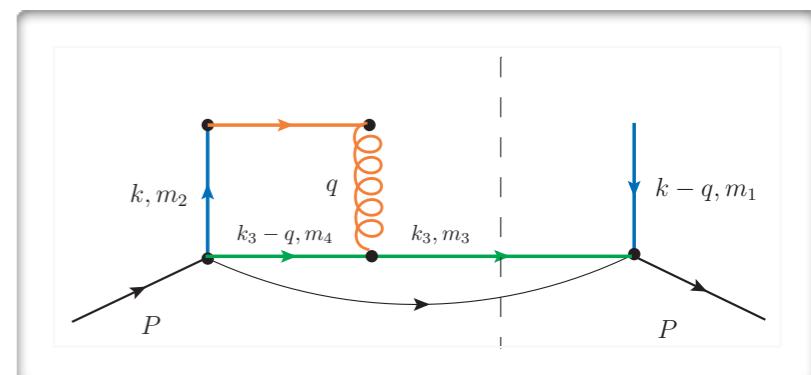
$$\mathcal{L}_{\vec{\xi}_T}(\infty, \xi^-) = \mathcal{P} \exp \left(-ig \int_{\xi^-}^{\infty} A^+(\eta^-, \vec{\xi}_T) d\eta^- \right)$$

- holds in covariant gauges
- process dependent
- explicit dependence on α_s

Final State Interactions in Hadronic Models

Twofold problem

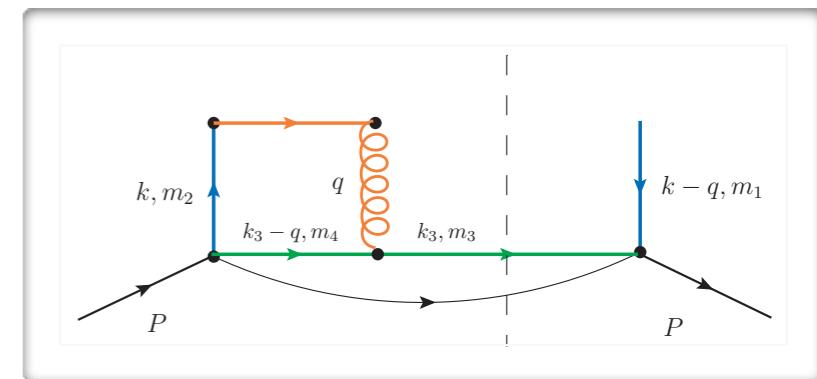
- FSI mimicked by a one-gluon-exchange
 - gluon propagator
- Explicit dependence on the coupling constant
 - relevance of NP scheme for model calculations



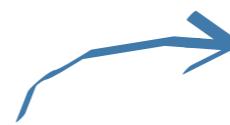
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Model calculations



F. Yuan, PLB 575
AC, Vento & Scopetta, PRD79 074001; PRD80 074032

- ♦ MIT bag model calculation
 - perturbative QCD governs the dynamics inside the confining region
 - no need for NP gluon propagator
 - NP scheme → change of hadronic scale

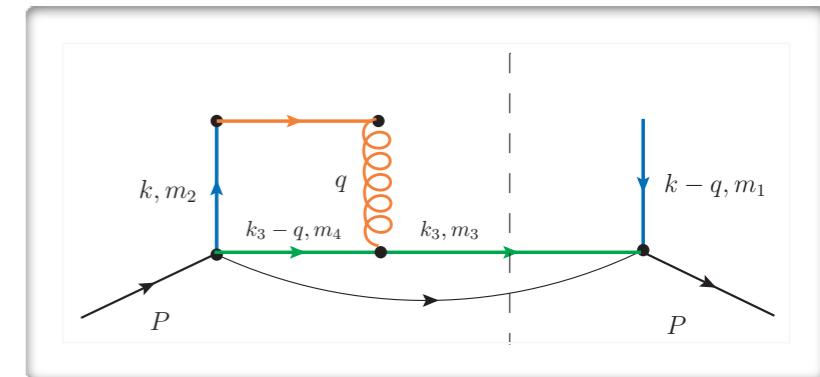
- ♦ Other model calculations?

e.g. L. Gamberg and M. Schlegel, Phys. Lett. B 685 (2010) 95

Final State Interactions in Hadronic Models

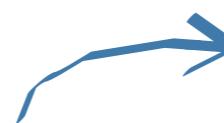
Twofold problem

- FSI mimicked by a one-gluon-exchange
 - **gluon propagator** ← **beyond the perturbative OGE approximation?**
- Explicit dependence on the coupling constant
 - **relevance of NP scheme for model calculations**



Model calculations

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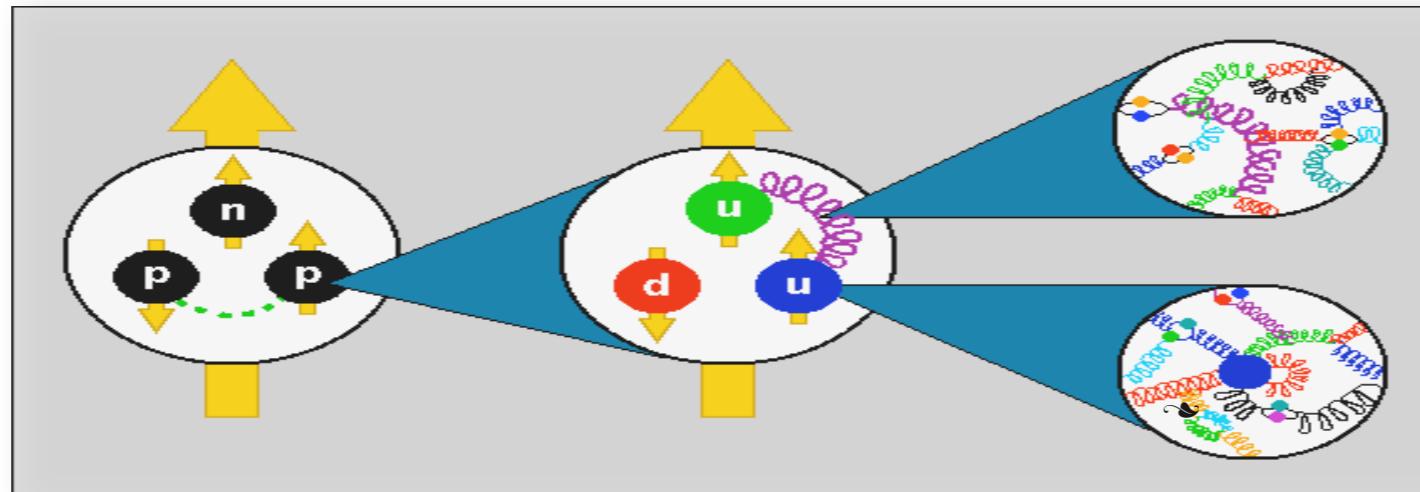
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Hadronic Models

Hadron \Leftrightarrow Constituent quarks \Leftrightarrow Current quarks



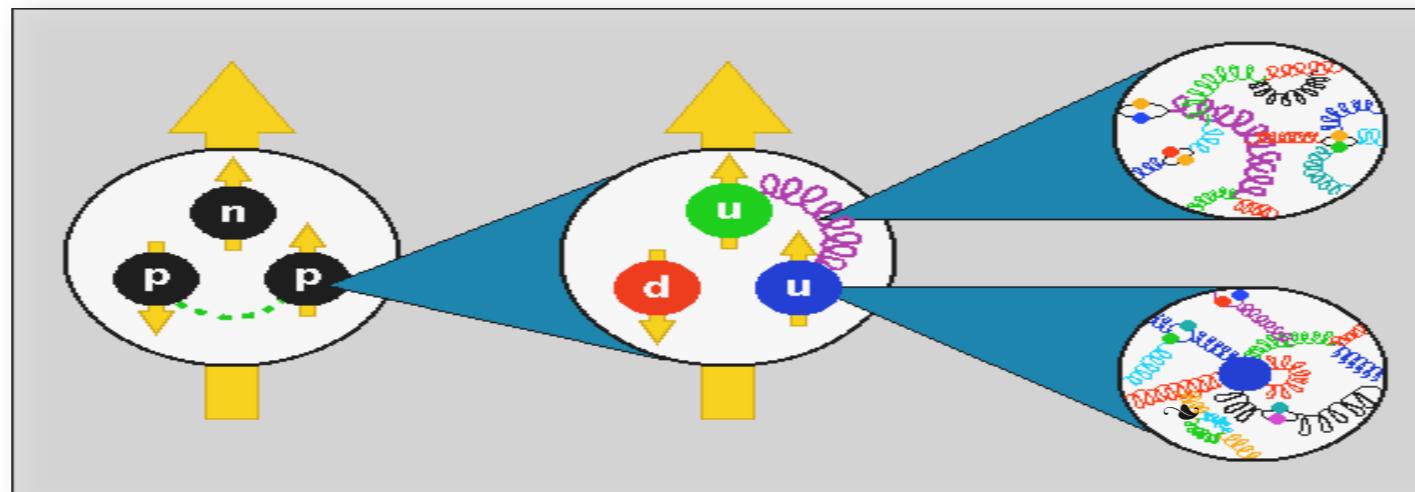
Nonperturbative vs. Perturbative QCD

Models of Hadron Structure

Renormalization Group Eqs.

Hadronic Models

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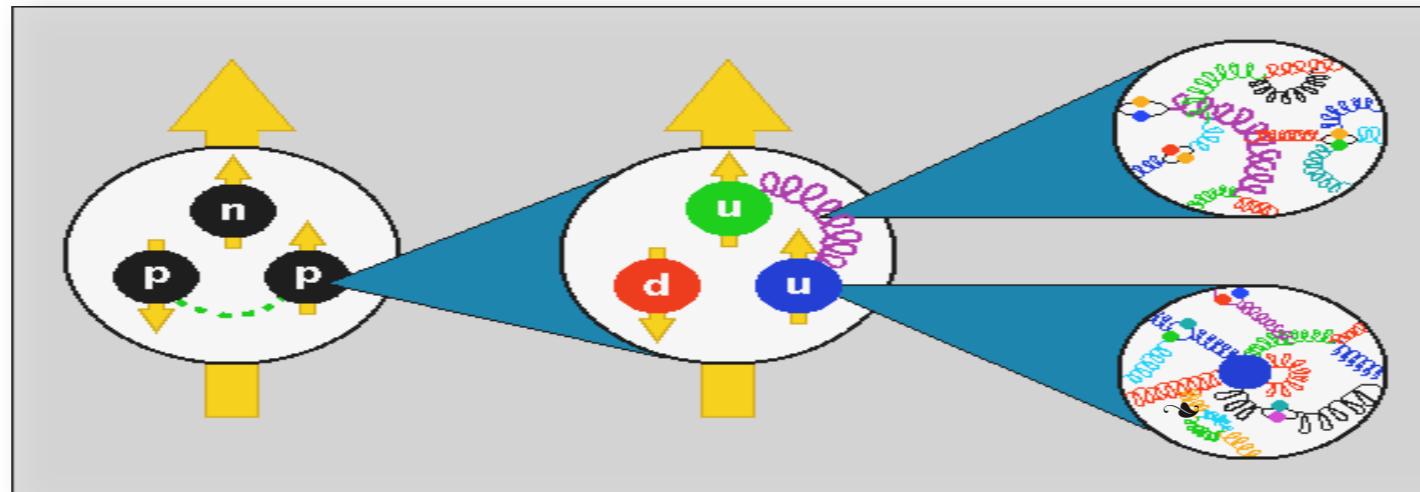
Renormalization Group Eqs.

Observable

- calculated in hadronic model
- at scale μ_0
- switch on QCD evolution

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Models of Hadron Structure

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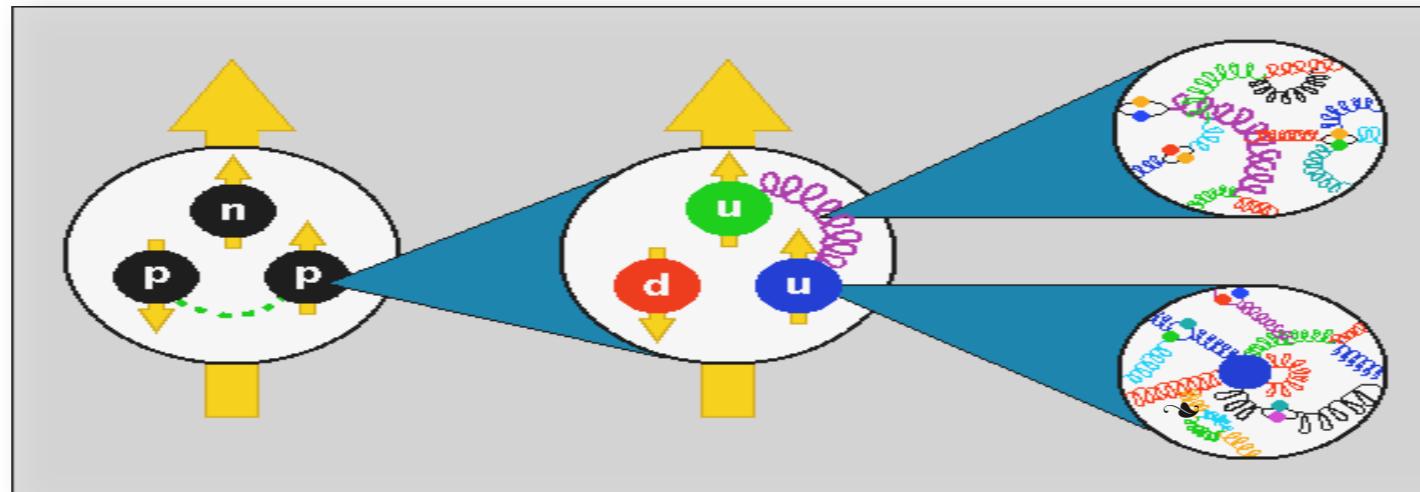
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Talk by Weiss

Nonperturbative vs. Perturbative QCD

Models of Hadron Structure

Renormalization Group Eqs.

Observable

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?

Hadronic Scale from collinear PDFs, e.g. CTEQ, GRV,...

We use RGE and one *first principle* based assumption.
Then we set scenarios ...

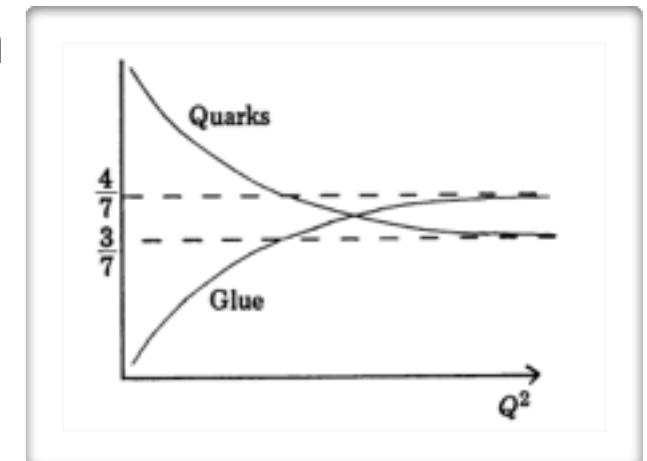
Hadronic Scale from collinear PDFs, e.g. CTEQ, GRV,...

We use RGE and one *first principle* based assumption.
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Say there exists a scale at which there is no sea and no gluon, then

$$\langle (u_v + d_v) (\mu_0^2) \rangle_{n=2} = 1$$

QCD evolution introduces gluons and sea quarks:



R.G.Roberts

"The Structure of the Proton"

$$\langle (u_v + d_v) (Q^2 = 10 \text{ GeV}^2) \rangle_{n=2} = 0.36$$



DATA= PDFs parameterization

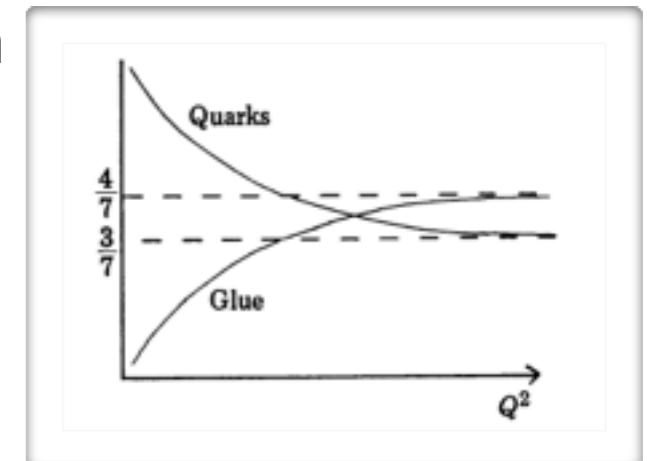
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DATA= PDFs parameterization

Evolve in energy until 2nd moment=1
Find $\mu_0^2 \sim 0.1 \text{ GeV}^2 + \Delta \mu_0^2$

Perturbative vs. NP ‘evolution’: Fixing the hadronic scale

2nd moment of f_1

$$\langle q_v(Q^2) \rangle_n = \langle q_v(\mu_0^2) \rangle_n \left(\frac{\alpha(Q^2)}{\alpha(\mu_0^2)} \right)^{d_{NS}^n}$$

L0 perturbative evolution

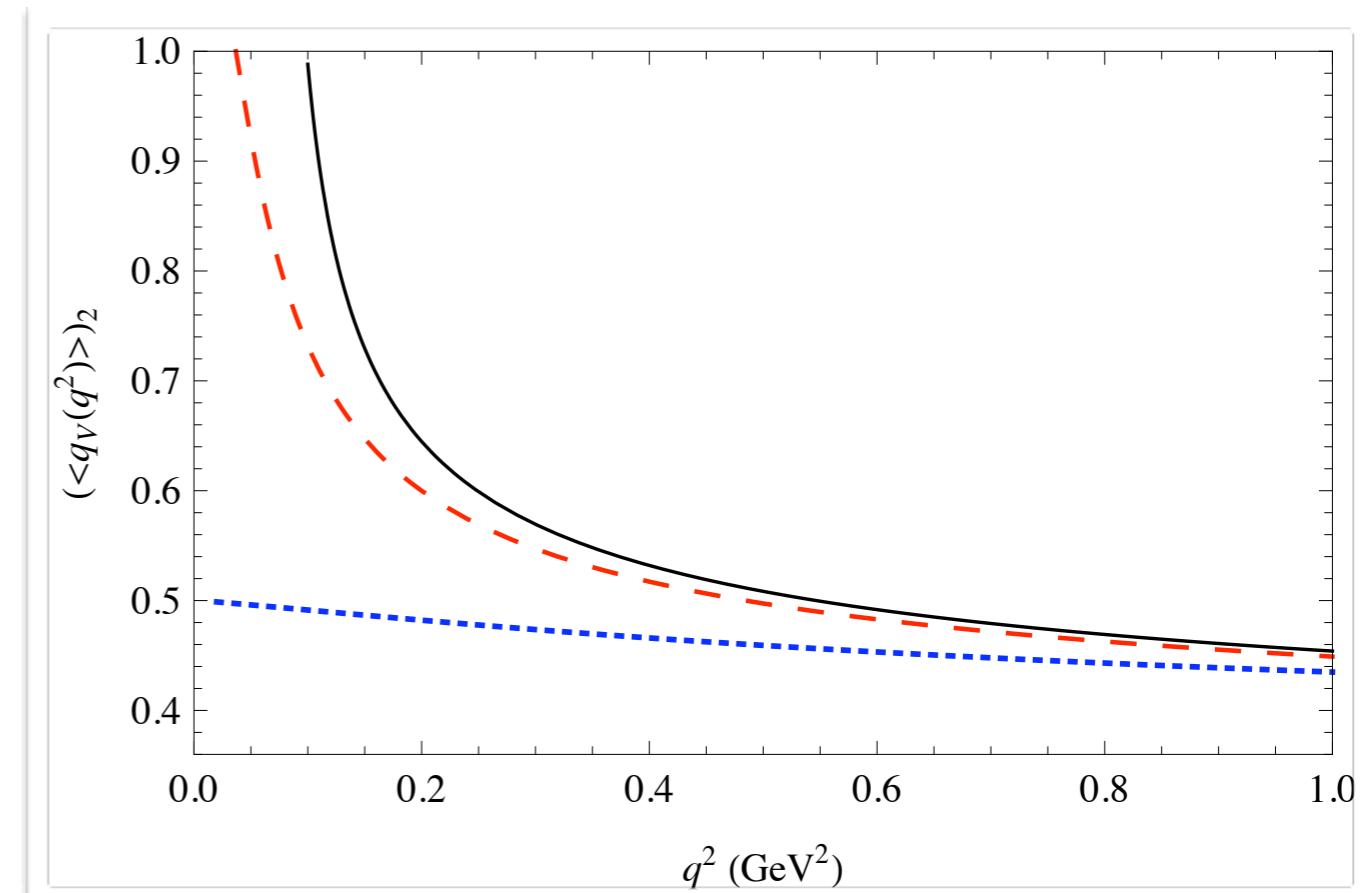
$\Lambda=250$ MeV ; \overline{MS} scheme

Low mass scenario NP coupling constant

$m_0=250$ MeV ; $\Lambda=250$ MeV ; $\rho=1.5$

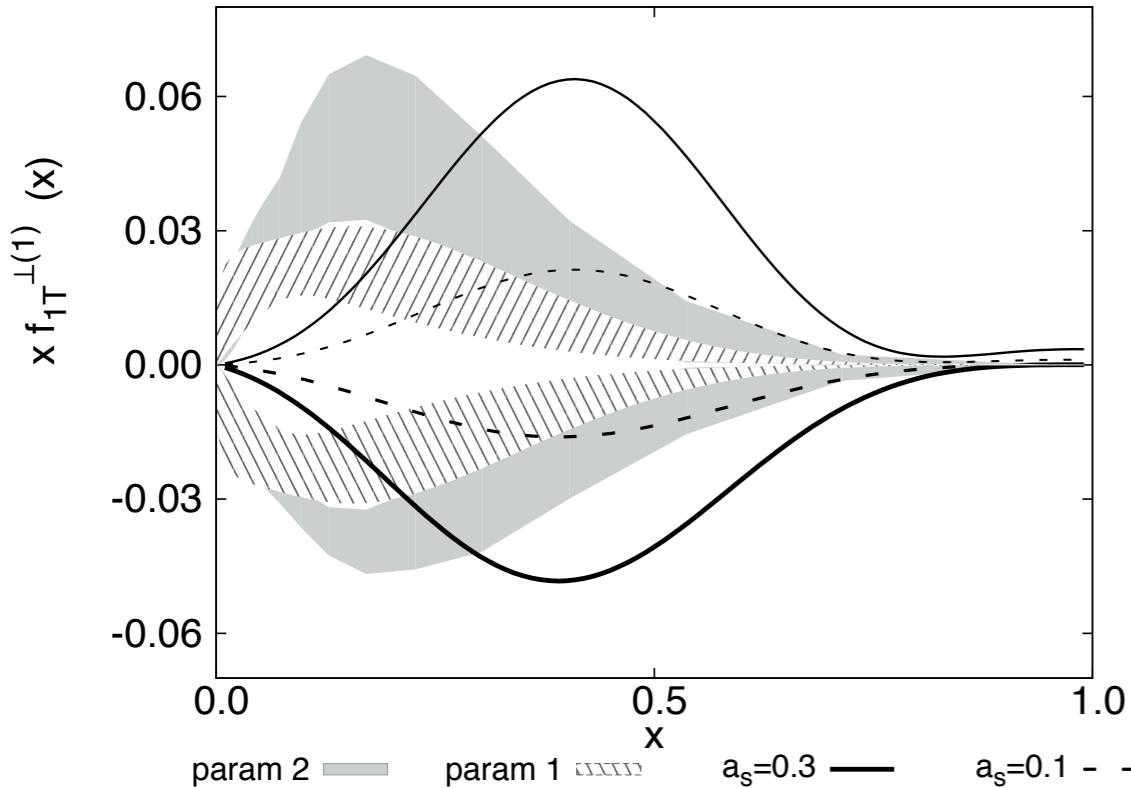
High mass scenario NP coupling constant

$m_0=500$ MeV ; $\Lambda=250$ MeV ; $\rho=2.$



Sivers & Boer-Mulders functions

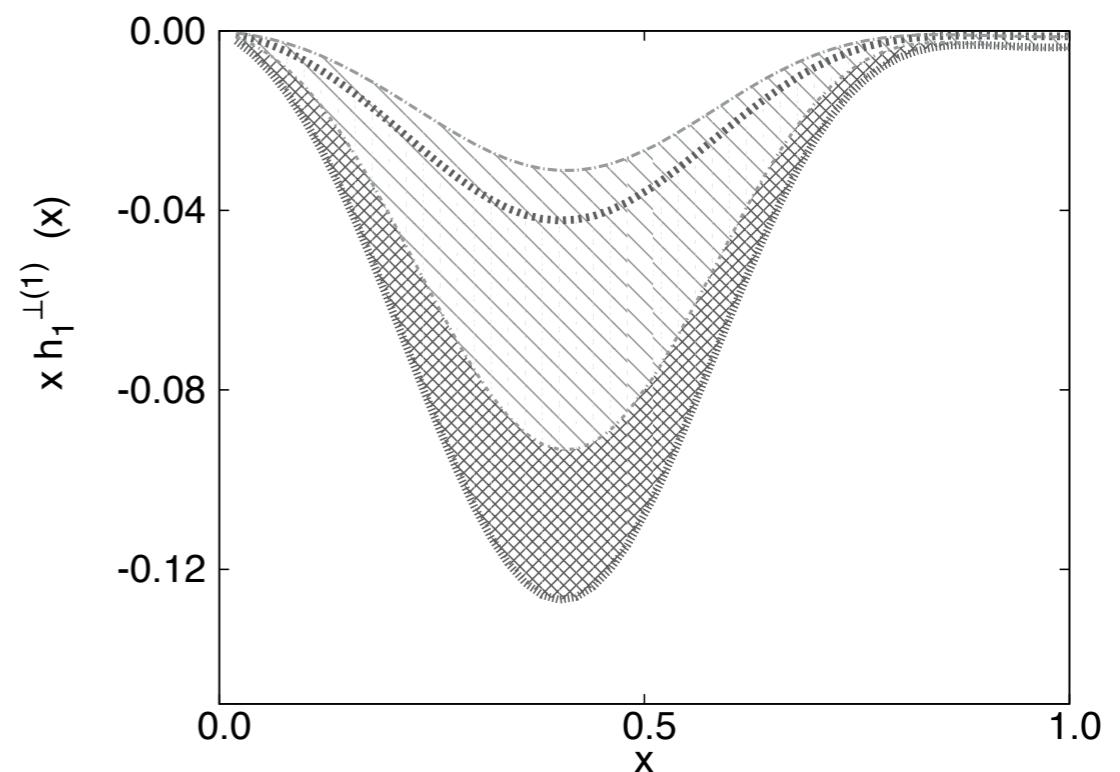
[A.C., Scopetta & Vento, Eur.Phys.J. A47]



$$0.1 < \frac{\alpha_s(\mu_0^2)}{4\pi} < 0.3$$

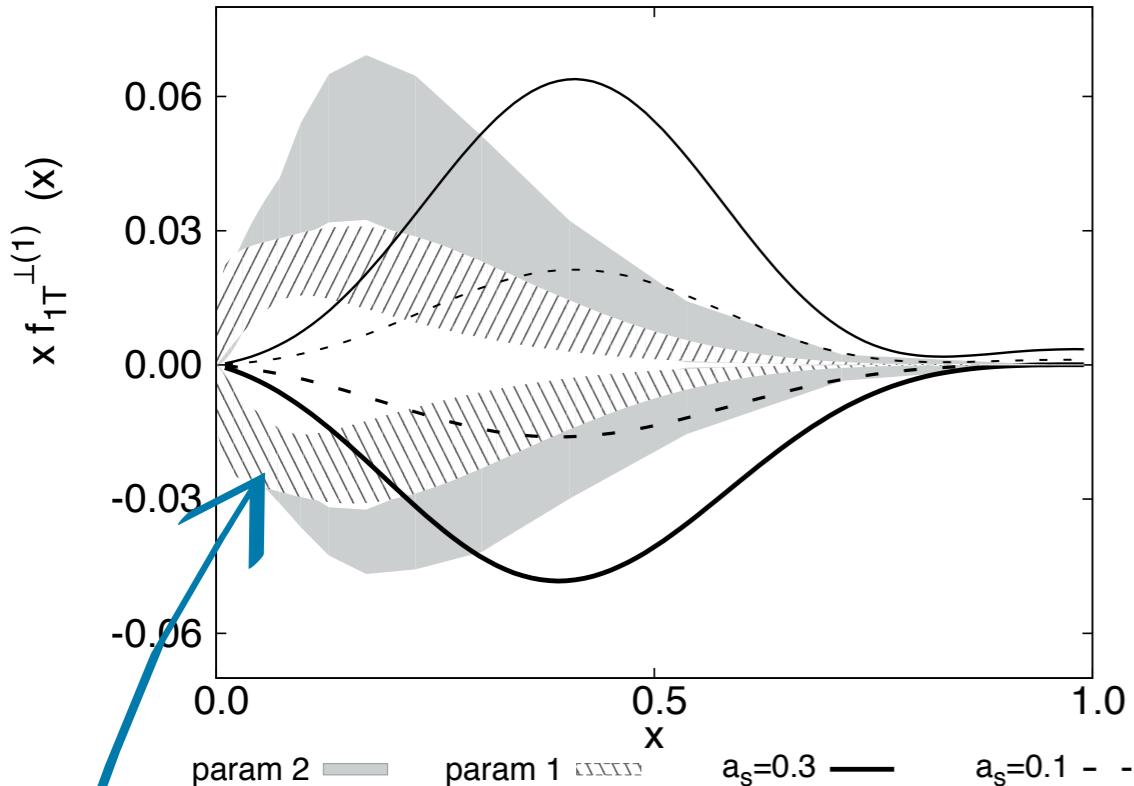
- dashed $\frac{\alpha_s(\mu_0^2)}{4\pi} = 0.1$
solid $\frac{\alpha_s(\mu_0^2)}{4\pi} = 0.3$

Bag Model:
rescaling/defining error
of f_{1T}^{\perp} & h_1^{\perp}



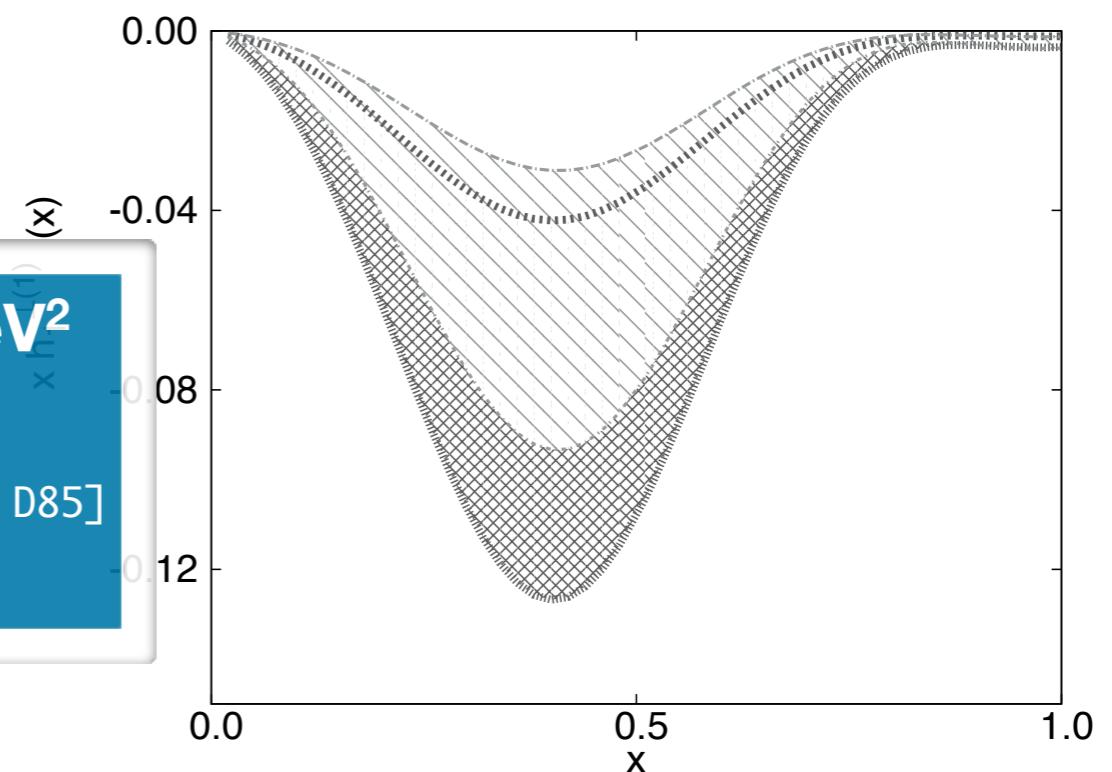
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dashed	$\frac{\alpha_s(\mu_0^2)}{4\pi} = 0.1$
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Phenomenological extractions at $Q^2=2.5\text{GeV}^2$

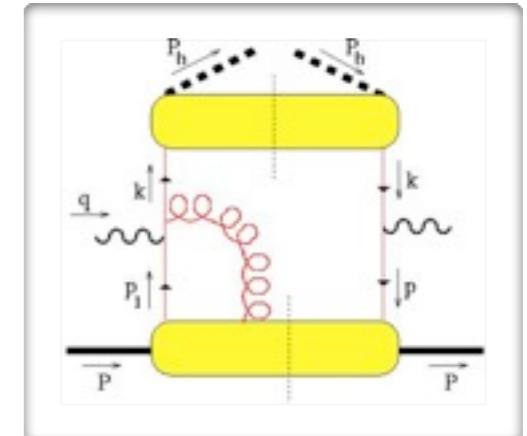
- Need for QCD formalism for T-odd TMDs
[Aybat, Collins, Qiu & Rogers, Phys.Rev. D85]
 - Additional source of error

Work in progress for T-odd TMDs

- Ambiguity Sivers function and Qiu-Sterman function

- Model dependent definition of the FSI and of the proton

Talk by Qiu



- TMD evolution: Coupled CSS and RGE -> two scales ! [Aybat et al., PRD85]

- Definition of momentum regions

[Bacchetta et al., JHEP 0808]

- Redefinition of both scales for model calculations (with T. Rogers)

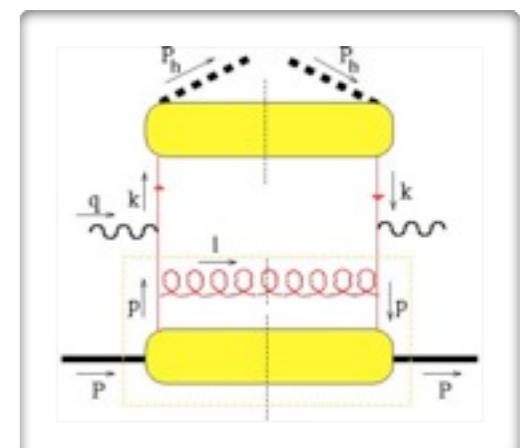
- Correspondance effective coupling from the soft blob with pQCD

- [Brodsky et al., Phys.Rev.D81] *À la Grunberg?* [Phys. Rev. D29]

- Commensurate Scale Relations

[Brodsky & Lu, Phys. Rev. D251]

Talk by Brodsky



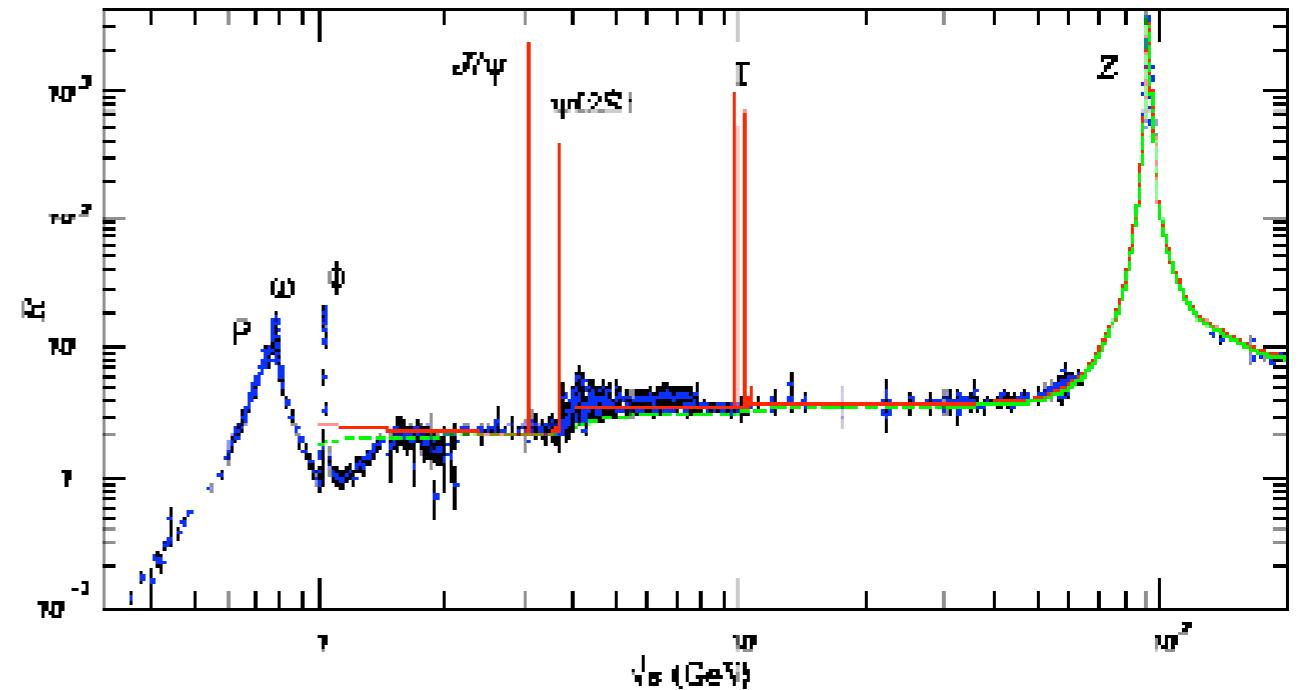
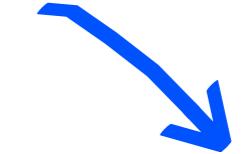
Hadron Structure Phenomenology

Parton-Hadron Duality

Parton-Hadron Duality

[Poggio, Quinn & Weinberg, Phys Rev D13]

$$e^+ - e^- \rightarrow \text{hadrons} \equiv \sum_q (e^+ e^- \rightarrow q\bar{q}) \Rightarrow \sigma_{\text{hadrons}} \equiv \sum_q \hat{\sigma}_q$$

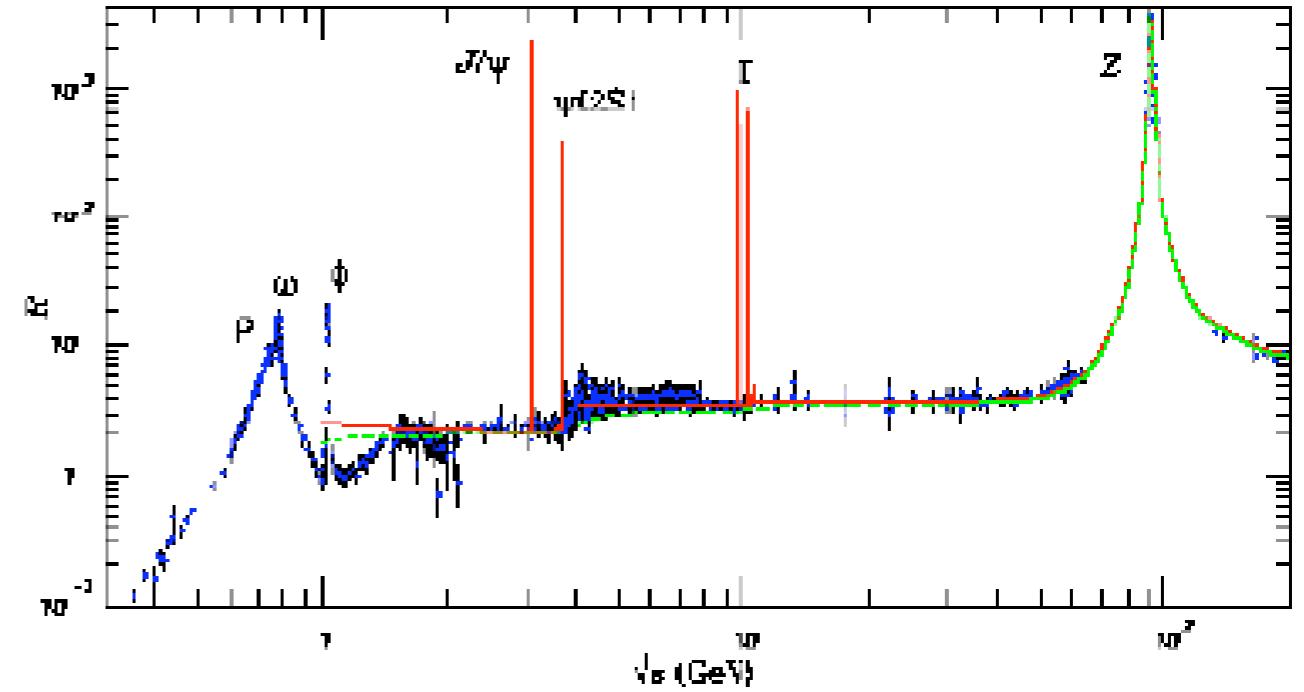
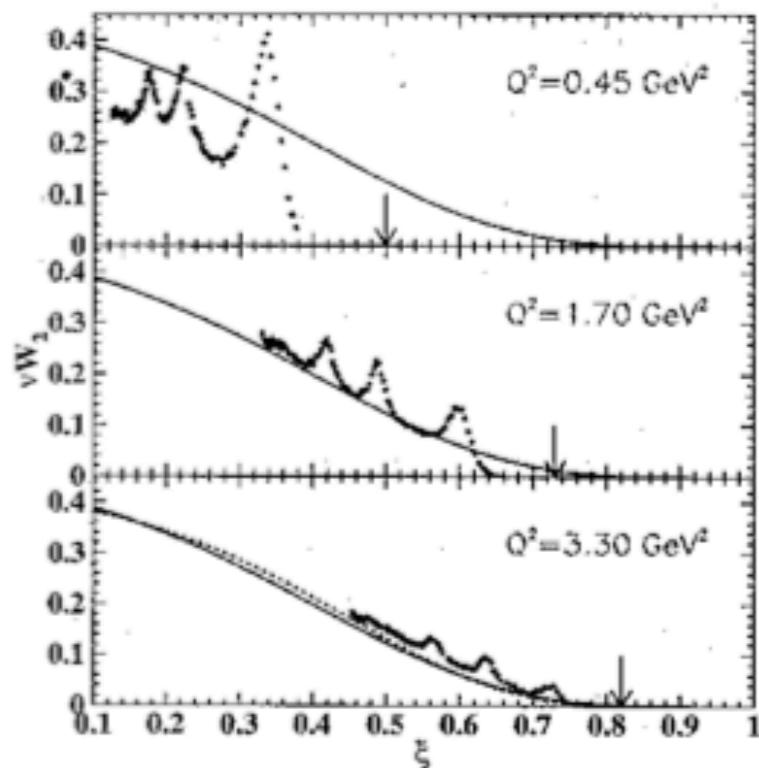


Complementarity between Parton and Hadron descriptions of observable

Parton-Hadron Duality

[Poggio, Quinn & Weinberg, Phys Rev D13]

$$e^+ - e^- \rightarrow \text{hadrons} \equiv \sum_q (e^+ e^- \rightarrow q\bar{q}) \Rightarrow \sigma_{\text{hadrons}} \equiv \sum_q \hat{\sigma}_q$$



Structure functions
Resonance region \Leftrightarrow Scaling region

$x_{Bj} > 0.5$, Q^2 multi-GeV region $\Rightarrow W^2 \leq 5 \text{ GeV}^2$

[Bloom & Gilman, Phys.Rev.Lett.25]

Complementarity between Parton and Hadron descriptions of observable

Two Complementary Approaches to Structure Functions

experiment

$$I^{res}(Q^2) = \int_{x_m}^{x_M} F_2^{\text{Res}}(x, Q^2) dx$$
$$I^{DIS}(Q^2) = \int_{x_m}^{x_M} F_2^{\text{DIS}}(x, Q^2) dx$$

perturbative QCD

$$x_M \div x_m \Leftrightarrow W^2_m \div W^2_M \Rightarrow 1 \div 4 \text{ GeV}^2$$

- Nonperturbative models analysis
- Perturbative analysis

Two Complementary Approaches to Structure Functions

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[Bianchi, Fantoni & Liuti, PRD69]

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- Nonperturbative models analysis
- Perturbative analysis

[Bianchi, Fantoni & Liuti, PRD69]

Start with NLO PDF and then ...

- Target Mass Corrections (TMC)
- Large-x Resummation (LxR)
- Higher-order in pQCD
- Higher-Twists

Two Complementary Approaches to Structure Functions

experiment

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[Bianchi, Fantoni & Liuti, PRD69]

Start with NLO PDF and then ...

- Target Mass Corrections (TMC) \longrightarrow 0k
- Large-x Resummation (LxR)
- Higher-order in pQCD
- Higher-Twists

[A. Accardi, J. -W. Qiu, JHEP 0807]
[A. De Rujula, et al., Phys. Lett. B64]

Two Complementary Approaches to Structure Functions

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perturbative QCD

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[Bianchi, Fantoni & Liuti, PRD69]

Start with NLO PDF and then ...

- Target Mass Corrections (TMC) \longrightarrow 0k
- Large-x Resummation (LxR) \longrightarrow pQCD
- Higher-order in pQCD
- Higher-Twists

[A. Accardi, J. -W. Qiu, JHEP 0807]
[A. De Rujula, et al., Phys. Lett. B64]

Two Complementary Approaches to Structure Functions

experiment

$$I^{res}(Q^2) = \int_{x_m}^{x_M} F_2^{\text{Res}}(x, Q^2) dx$$
$$I^{DIS}(Q^2) = \int_{x_m}^{x_M} F_2^{\text{DIS}}(x, Q^2) dx$$

$$x_M \div x_m \Leftrightarrow W^2_m \div W^2_M \Rightarrow 1 \div 4 \text{ GeV}^2$$

perturbative QCD

- Nonperturbative models analysis
- Perturbative analysis

[Bianchi, Fantoni & Liuti, PRD69]

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Large-x Resummation

Text Book, e.g. *Cornerstones of QCD*, M. Pennington.

- Large invariants: $\Lambda^2 \ll W^2 \ll Q^2$
- Argument for α_s is ω^2 , mass square of final state of γ^* parton collision

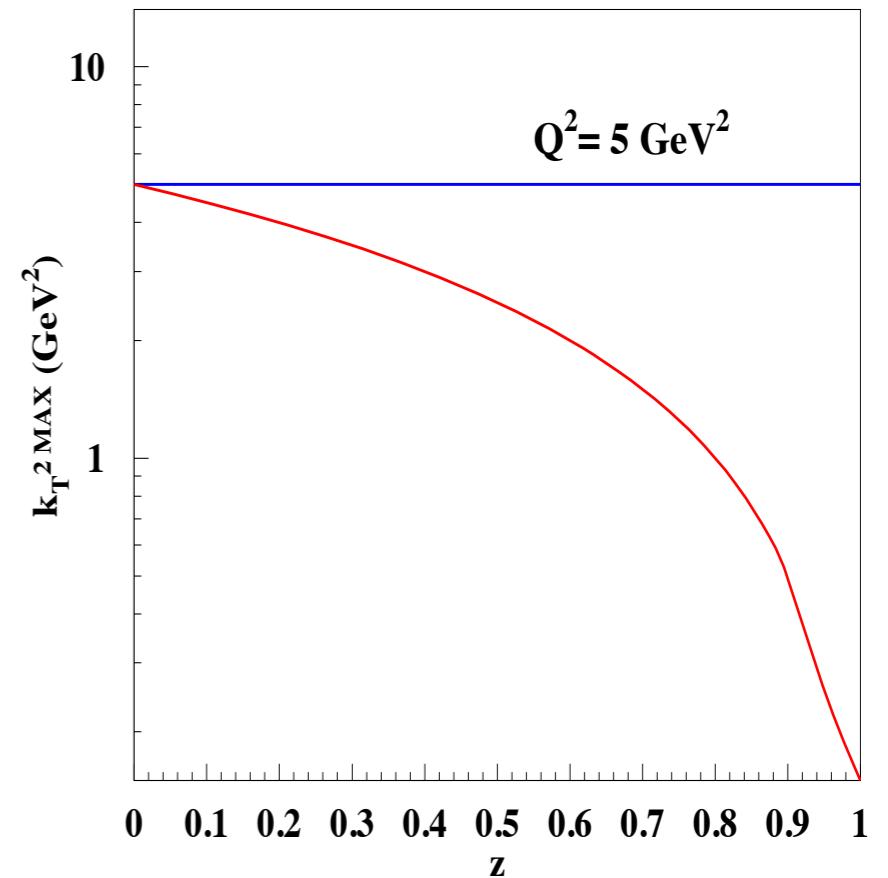
$$\omega^2 = \frac{Q^2}{z} (1-z)$$

Without LxR, upper limit $= Q^2$

$$q(x, Q^2) = \int_x^1 \frac{dz}{z} \int_{\mu^2}^{Q^2 \frac{1-z}{4z}} dk_T^2 \alpha_S(k_T^2) P_{qq}(z) q\left(\frac{x}{z}, k_T^2\right)$$

The structure functions become

$$F_2^{NS}(x, Q^2) = \sum_q \int_x^1 dz \frac{\alpha_s\left(\frac{Q^2(1-z)}{4z}\right)}{2\pi} C_{NS}(z) q_{NS}\left(\frac{x}{z}, Q^2\right)$$



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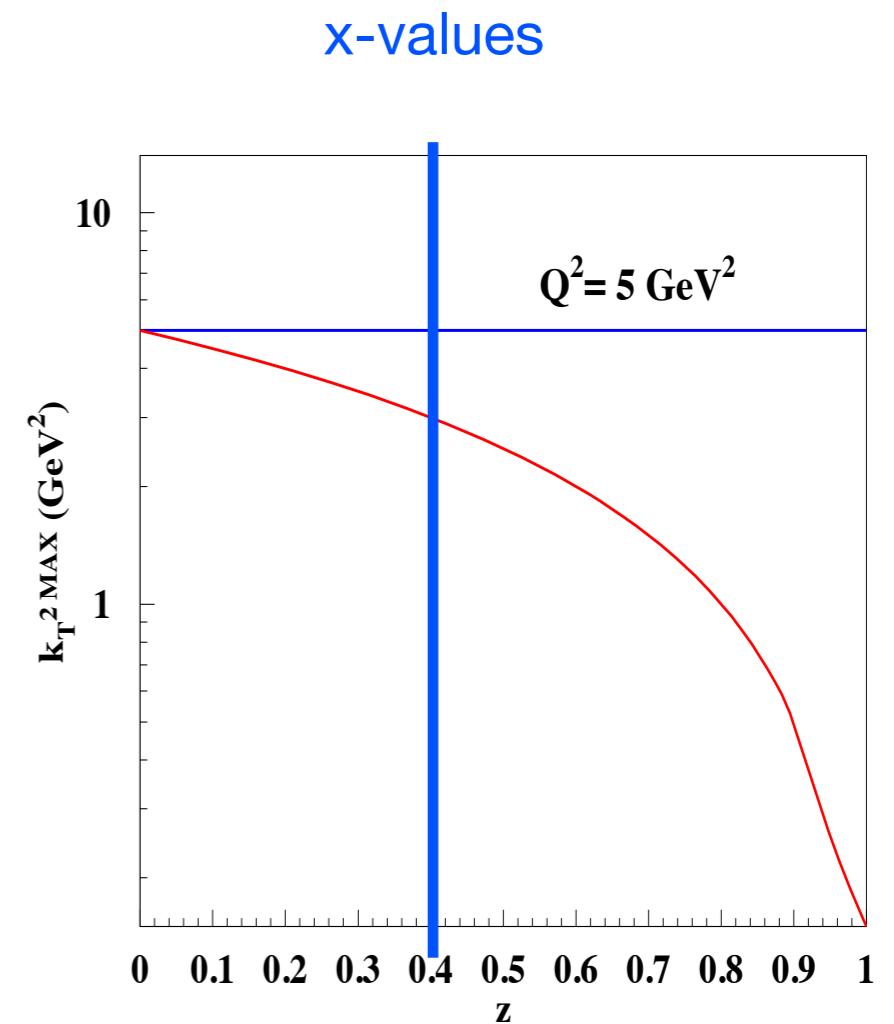
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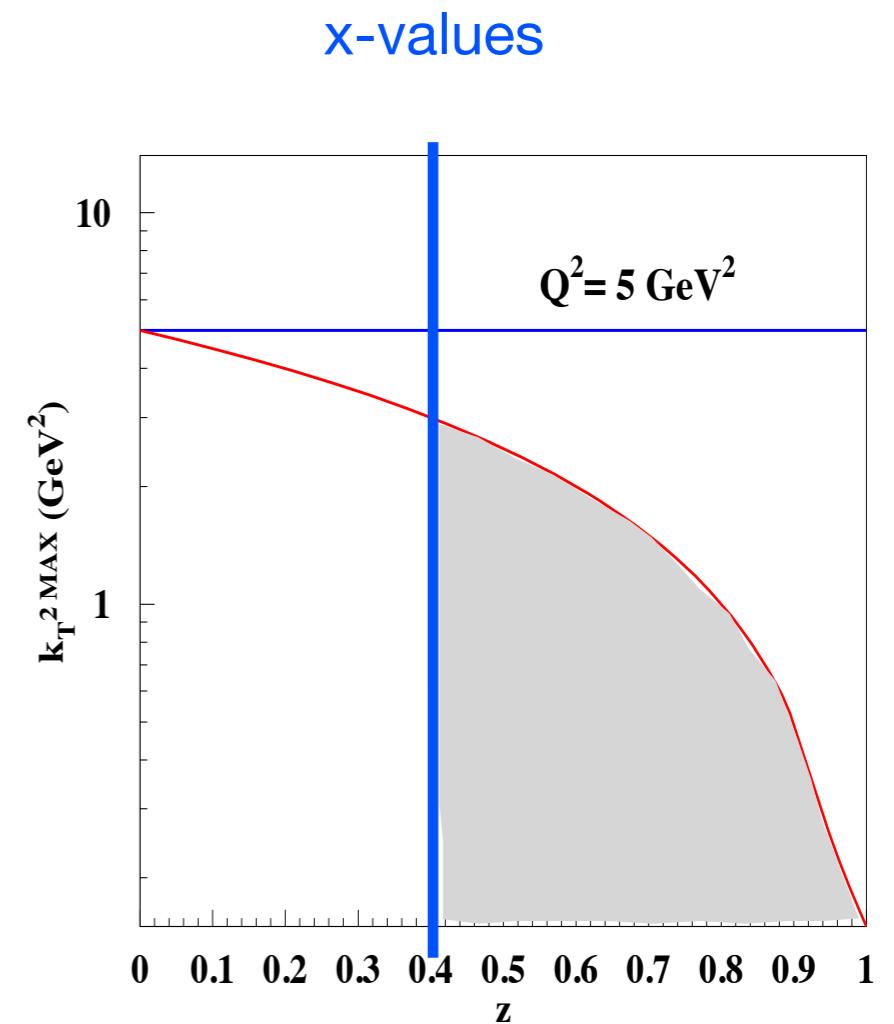
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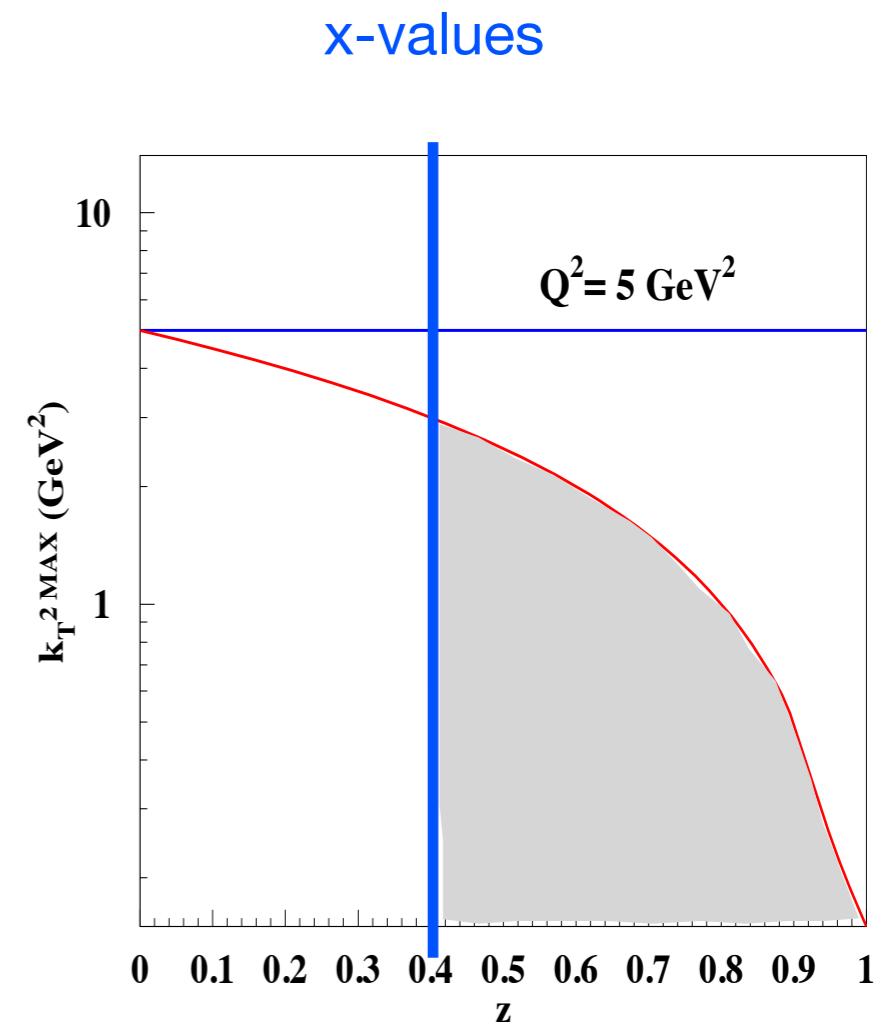
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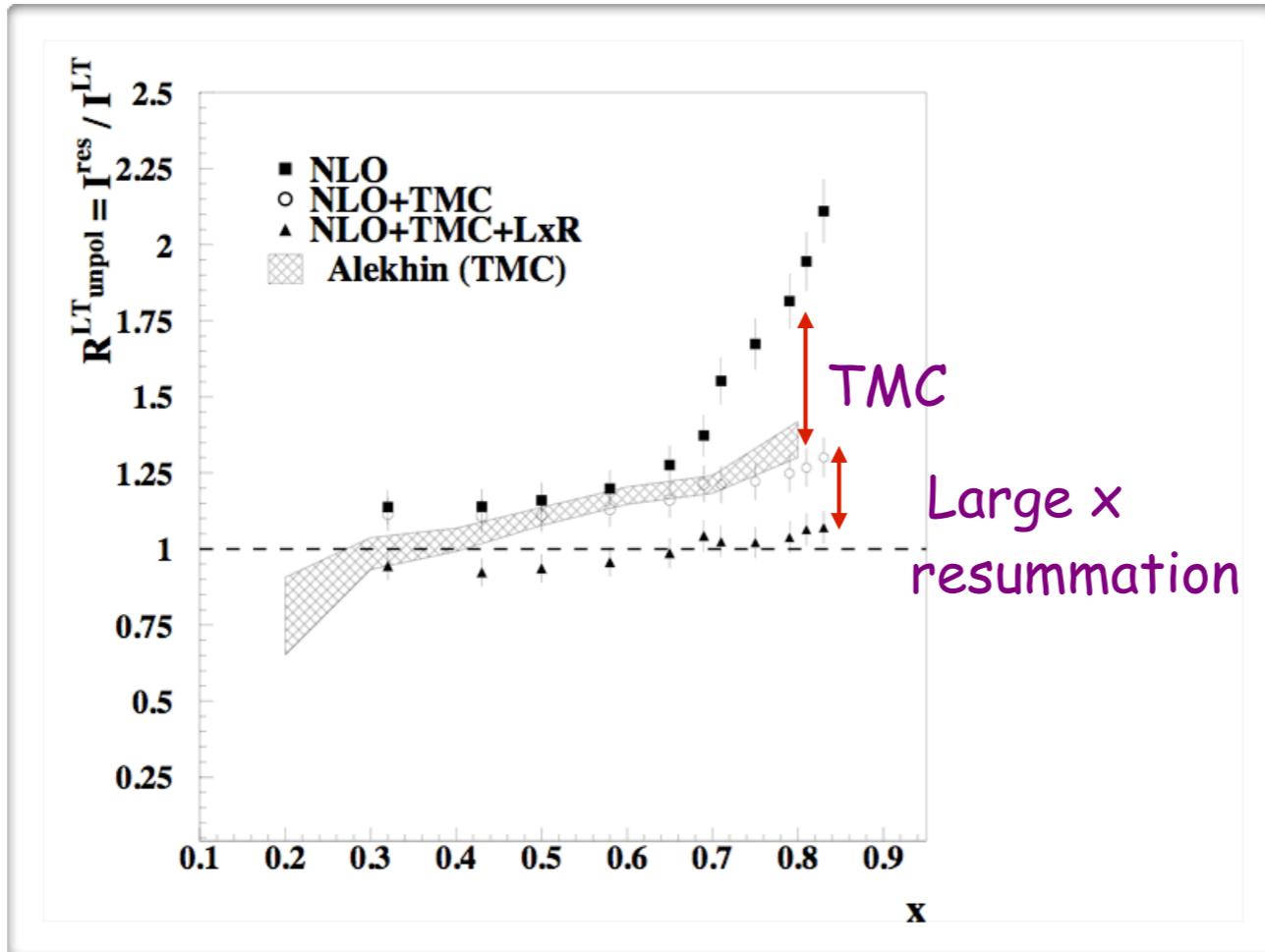
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What happens when $\Lambda^2 \sim W^2 \ll Q^2$?

Size of Nonperturbative Contributions

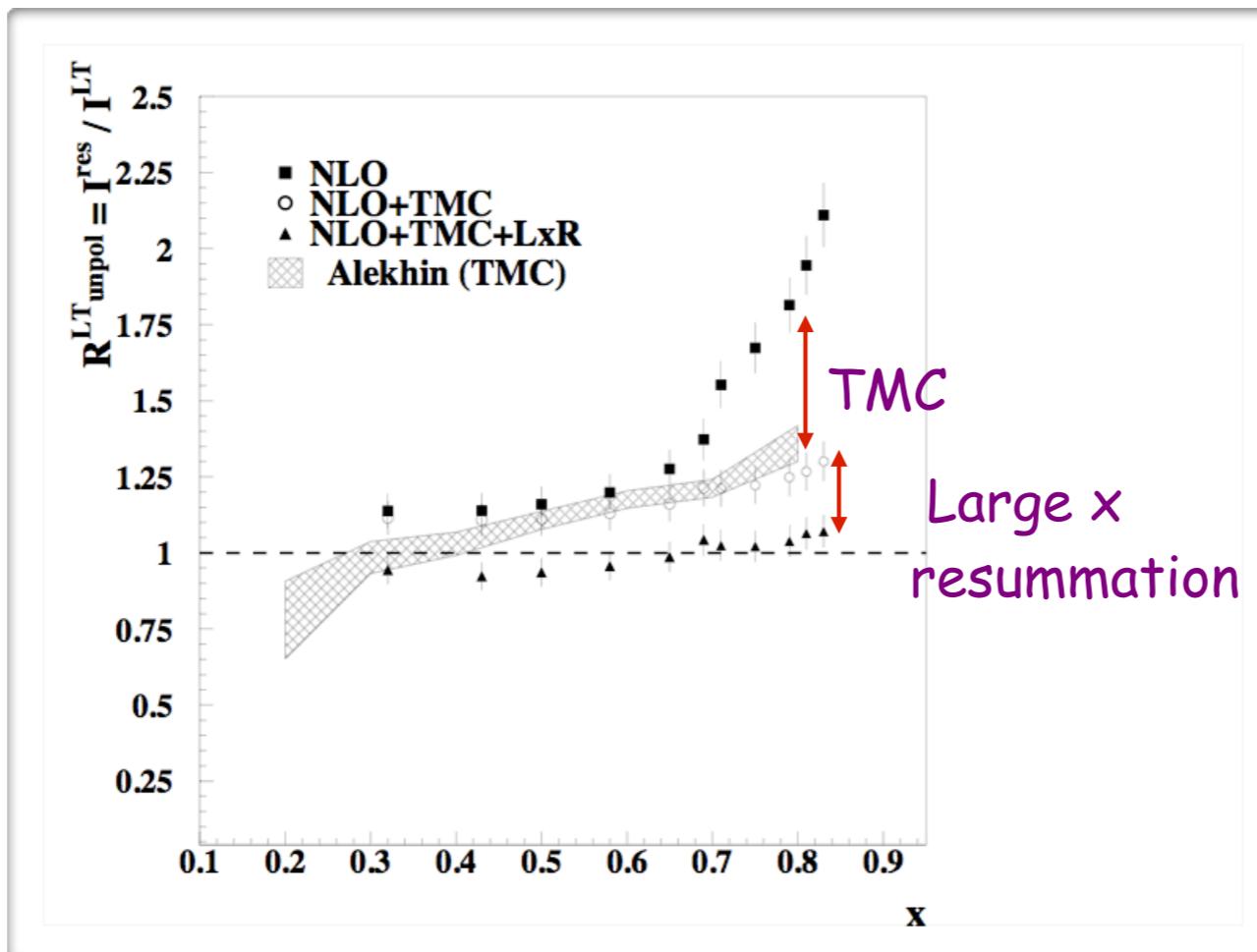


[Niculescu et al., PRD60]

[Bianchi, Fantoni & Liuti, PRD69]

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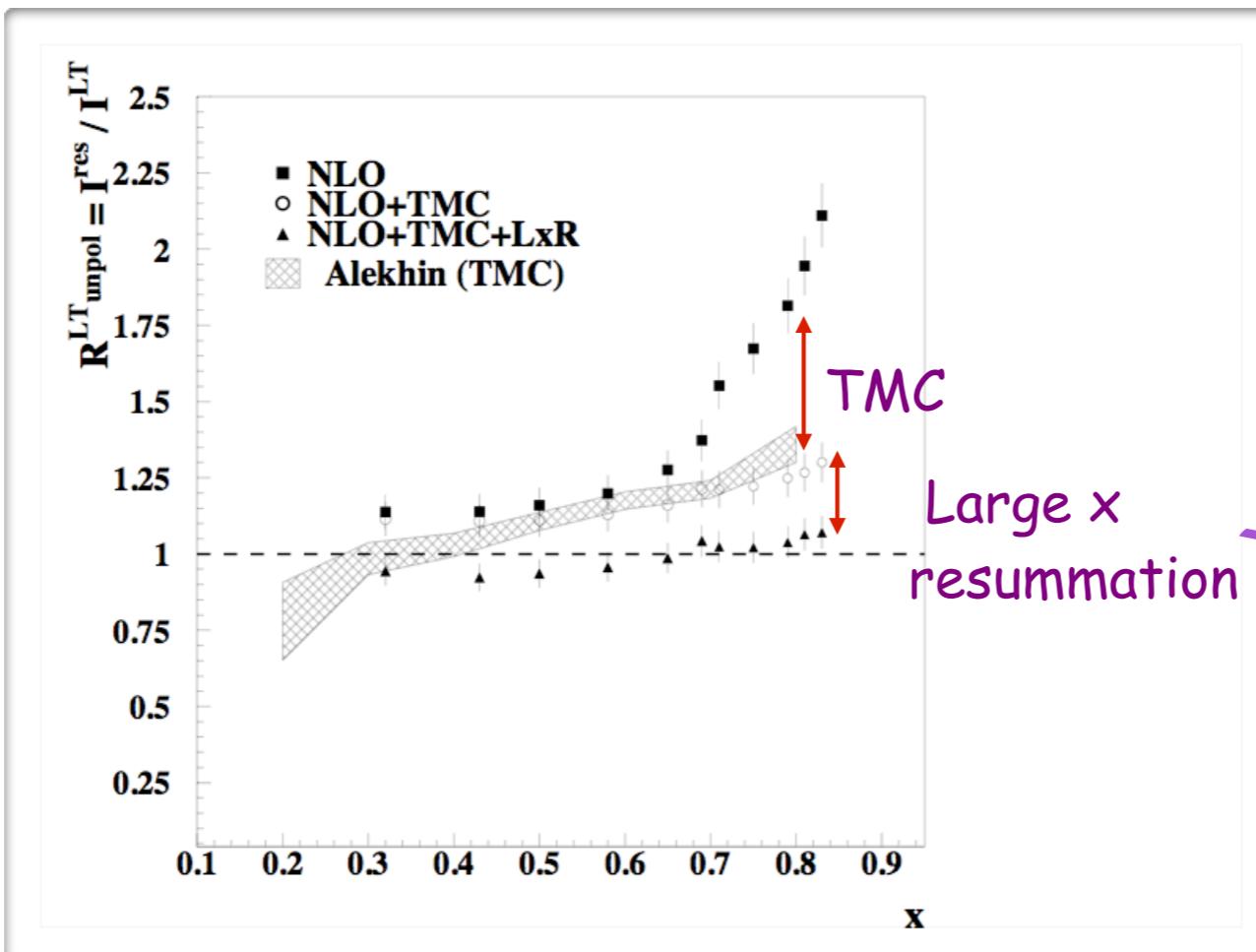


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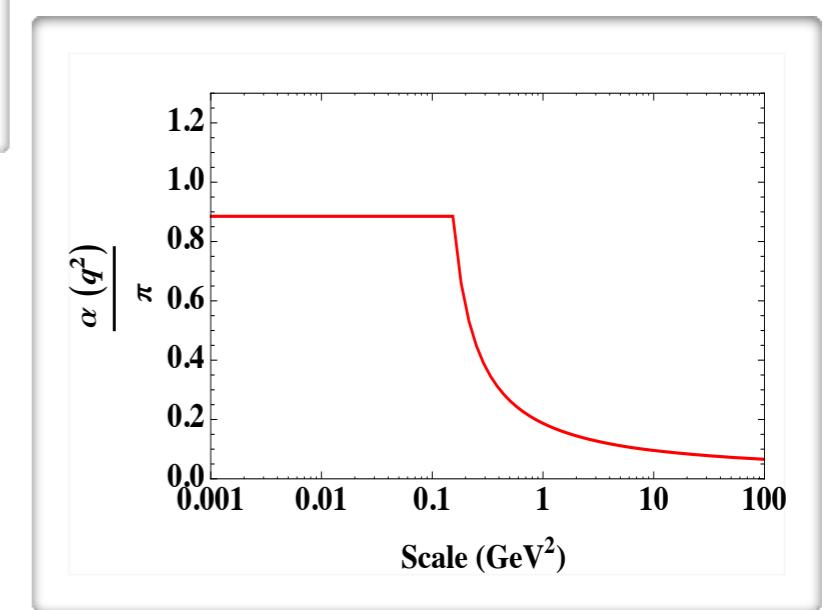


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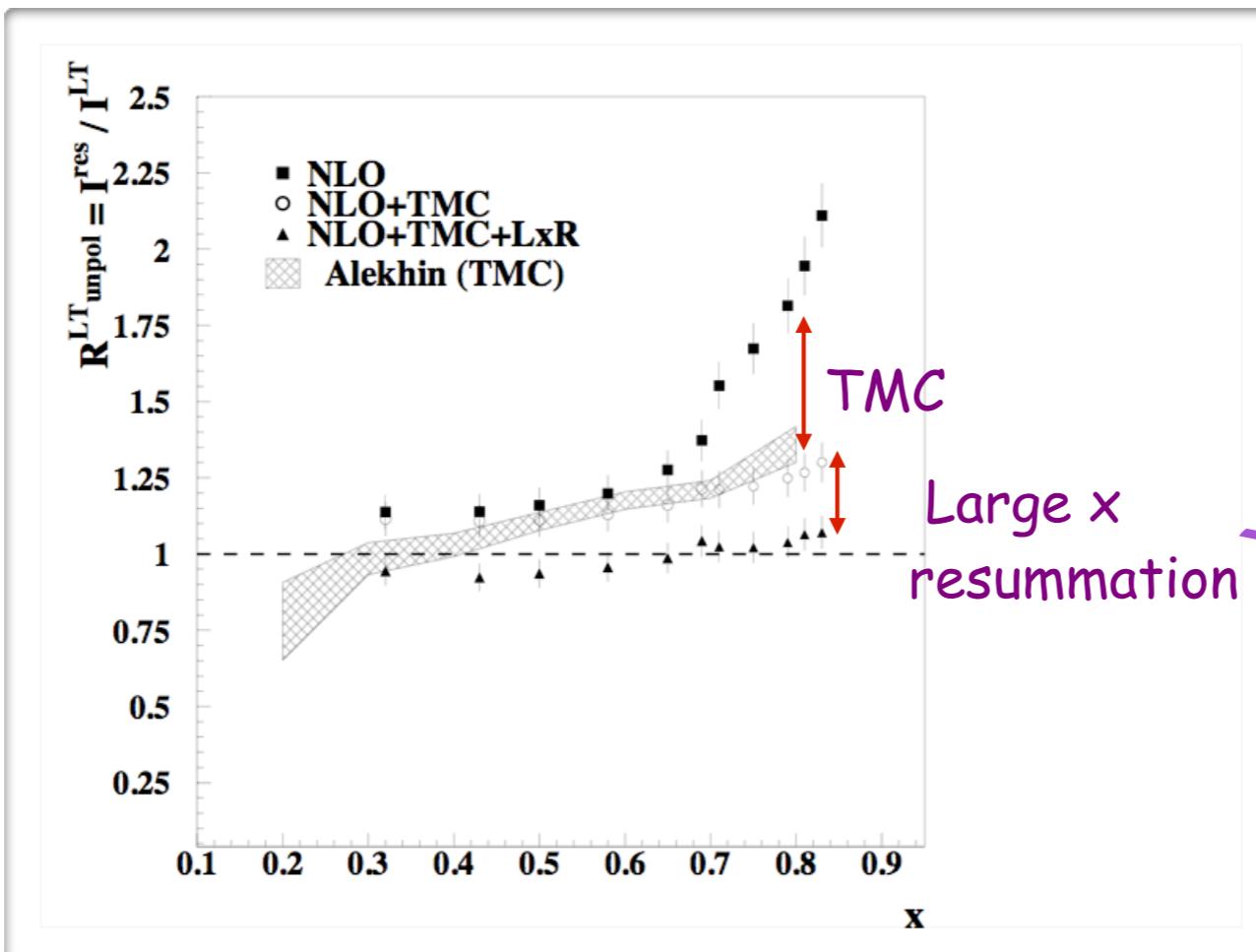
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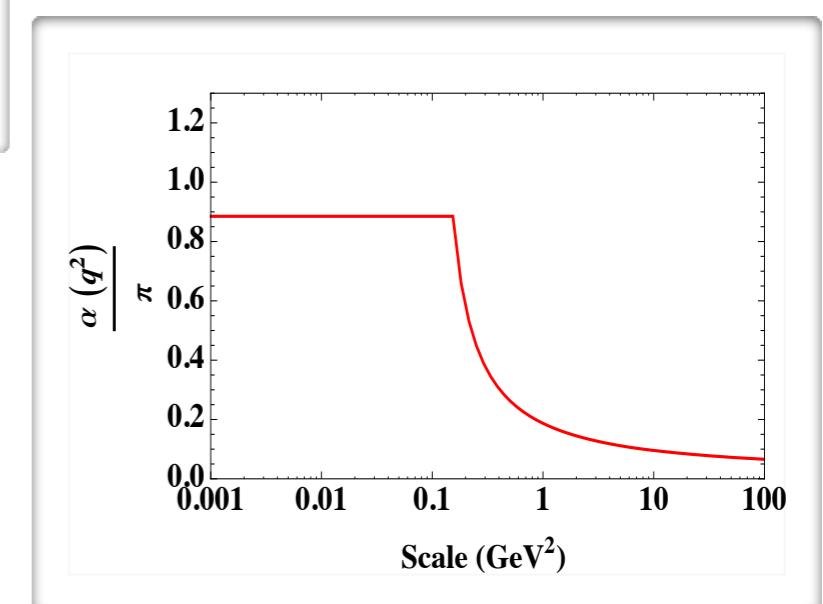


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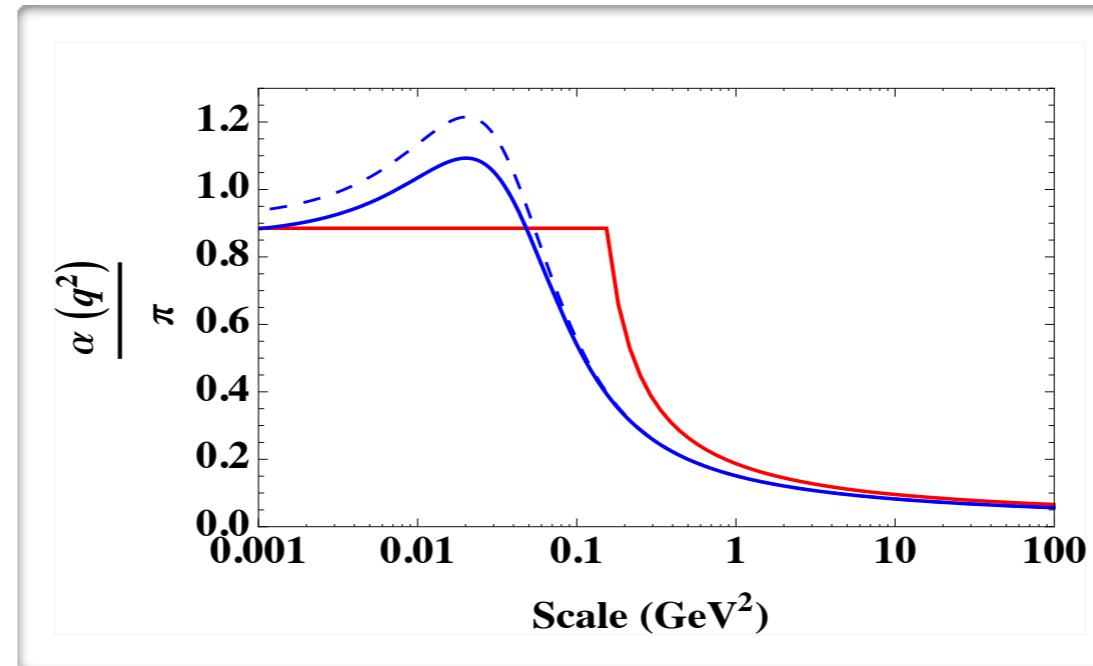
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New JLab data to be analyzed (P. Monaghan)

Nonperturbative Coupling Constant & LxR

How we go further : Nonperturbative Coupling Constant from DSE



Cornwall α_s^{NP}

3-4 free parameters

(up to physical constraints)

- Nonperturbative effects gathered in effective coupling α_s^{NP}
- Use of NP running coupling that scales to LO pQCD result
- Include in LxR
- Parameterization of the realization of duality
- Understand Higher-Twists ?
- Go to NNLO ?

Work in progress with S. Liuti

Nonperturbative QCD coupling from Phenomenology

Joint analysis: Chen, Courtoy, Deur, Liuti & Vento

Work in progress about α_s at low energy

- **Nonperturbative to perturbative transition**
 - Final States Interactions and pQCD
 - Errorbands to measurements (even if error on “model dependence” is immeasurable)
- **Perturbative to nonperturbative transition**
 - Realization of duality & parametrization via α_s^{NP}
 - New data for F_2 in the resonance region at JLab
- **How to relate the coupling constant?**
 - Commensurate Scale Relations? [Brodsky & Lu, Phys. Rev. D251]
 - RG-improved perturbation theory? [Grunberg, Phys. Rev. D29]

Extraction of α_s at low energy

- Polarized scattering from both proton and neutron

Deur et al. Phys.Lett. B650 (2007) 244-248

Natale, PoS QCD-TNT09 (2009) 031

Bjorken Sum Rule from JLab & GDH Sum Rule at $Q^2=0$ GeV 2

- Deep Inelastic Scattering (DIS) at large Bjorken-x & parton-hadron duality

Liuti, [arXiv:1101.5303 [hep-ph]].

- Semi-Inclusive DIS & Extraction of T-odd TMDs from SSAs

A.C., Vento & Scopetta, Eur. Phys. J. A47, 49 (2011)

Joint analysis: Chen, Courtoy, Deur, Liuti & Vento

Back-up Slides

Target Mass Corrections

$$F_2^{LT(TMC)}(x, Q^2) = \frac{x^2}{\xi^2 \gamma^3} F_2^\infty(\xi, Q^2) + 6 \frac{x^3 M^2}{Q^2 \gamma^4} \int_\xi^1 \frac{d\xi'}{\xi'^2} F_2^\infty(\xi', Q^2),$$

Accardi & Qiu :

$$F_{T,L}(x_B, Q^2, m_N^2)$$

$$= \int_\xi^{\xi/x_B} \frac{dx}{x} h_{f|T,L}(\tilde{x}_f, Q^2) \varphi_f(x, Q^2). \quad (18)$$

Instead of 1

Large-x Resummation

As a consequence...

$$\alpha_s(Q^2) \rightarrow \alpha_s[Q^2(1-z)] \approx \alpha_s(Q^2) - \frac{1}{2}\beta_0 \underline{\ln(1-z)} (\alpha_s(Q^2))^2$$

This takes care of the large log term in the Wilson coefficient f .
(NLO, MS-bar)

$$\left\{ \begin{array}{l} F_2^{NS}(x, Q^2) = \frac{\alpha_s}{2\pi} \sum_q \int_x^1 dz \cancel{C_{NS}(z)} q_{NS}(x/z, Q^2), \\ C_{NS}(z) = \delta(1-z) + \left\{ C_F \left(\frac{1+z^2}{1-z} \right)_+ \left[\ln \left(\frac{1-z}{z} \right) - \frac{3}{2} \right] + \frac{1}{2} (9z+5) \right\} \end{array} \right. \quad (24)$$

The scale that allows one to annihilate the effect of the large $\ln(1-z)$ terms at large x at NLO is the invariant mass, W^2

Equivalent to a resummation of these terms up to NLO